Financial Economics

Interest Rates

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Student version

Learning Objectives

Understand the different ways interest rates are quoted

- Use quoted rates to calculate loan payments and balances

Know how interest rates are determined

- How expectations, inflation and risk combine to determine interest rates?

What discount rate should we use to discount cash flows from a project?

- **The *Opportunity Cost of Capital* (OCC)**

Interest rate: a **return** rate for the lender *versus* a **cost** for the borrower

Interest rate markets are among the largest capital markets in the world

- **Money Markets**: Short term
- **Bond Markets**: Medium and Long term

In practice, interest rates are quoted in different ways

- **Account types: Livret A** : 2.25% ; Livret d'épargne populaire (**LEP**) : 2.5% (February 2012)
- **Banks:** a savings account at LCL offers 1.6% interest rate ; 2.5% at ING direct
- **Investment horizon:** French Treasury Bills (BTF) - 3 months : 0.6% ; Obligation assimilable du Trésor OAT - 10 years:3% (February 2012)
- **Risk and the identity of the borrower:** The French Government is able to borrow at a lower interest rate than Sanofi-Aventis, which in turn can borrow at a lower rate than a Small and medium enterprise (SME)

How to determine the appropriate discount rate for a given stream of cash flows, according to the investment horizon and the risk of default ?

Adjusting the interest Rate to Different Time Periods

Earning a 10% return annually is **not** the same as earning 5% every six months.

True ☐
False ☐

		Jan./01	June/31	Dec./31	Total return at the end of the year
Annual return of 10%	**Original**	**€10,000**		**€10,000**	**€11,000**
	Interest		**€0**	**€1,000**	
Semestrial return of 5%	**Original**	**€10,000**	**€10,000**	**€10,000**	**…**
	Interest		**€500**	**€500**	
Semestrial return of 5%	**Original**	**€10,000**	**€10,000**	**€10,500**	**…**
	Interest		**€500**	**…**	

***without* the compounding effect**

***with* the compounding effect**

General Equation for Rate Period Conversion

$$Equivalent\ t\ period\ rate = (1 + r)^t - 1$$

The equivalent Annual rate of a semestrial rate of 5% is:

Equivalent Annual rate = ……..

Note: *t* = 2 since we are solving for the annual (or 2 semesters) rate

Adjusting the interest Rate to Different Time Periods

Quick-Check Problem

Your bank account pays interest monthly with the interest rate quoted as an effective annual rate (EAR) of 6%. What amount of interest will you earn each month? If you have no money in the bank today, how much will you need to save at the end of each month to accumulate €100,000 in 10 years?

Effective Annual Rate (EAR) *versus* Annual Percentage Rate (APR)

The **annual percentage rate - APR**

Indicates the amount of simple interest earned in one year.

Simple interest is the amount of interest earned *without* the effect of compounding.

$$\textit{Interest rate per Compounding Period} = \frac{\text{APR}}{k}$$

The **Effective Annual Rate – EAR** or Effective Annual Yield (EAY)

Indicates the total amount of interest that will be earned at the end of one year

Considers the effect of compounding

$$\textit{Equivalent t period rate} = (1 + \text{r})^{t} - 1$$

$$\Rightarrow \quad EAR = \left(1 + \frac{\text{APR}}{k}\right)^{k} - 1$$

Effective Annual Rate (EAR) *versus* Annual Percentage Rate (APR)

Example: Alain Tauxvabien suggests you invest €10,000 in an account paying interests every month. The interest rate quoted as an APR is 6%. What amount of interest will you earn at the end of the year without the compounding effect? With the compounding effect? How much will you have in the account in 1 year?

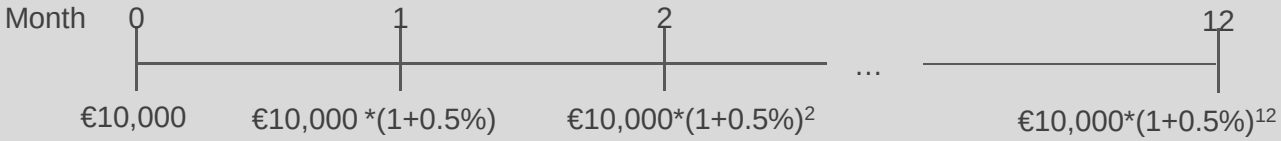
1 *Without* the compounding effect

➡

➡ Total return= …

➡

2 *With* the compounding effect



➡

➡ Total return = …

➡

Effective Annual Rate (EAR) *versus* Annual Percentage Rate (APR)

Effective Annual Rates for a 6% APR with Different Compounding Periods

Compounding Interval	*K*	Effective Annual Rate
Annual	1	$(1 + 0.06/1)^1 - 1 = 6\%$
Semiannual	2	$(1 + 0.06/2)^2 - 1 = 6.09\%$
Monthly	12	$(1 + 0.06/12)^{12} - 1 = 6.1678\%$
Daily	365	$(1 + 0.06/365)^{365} - 1 = 6.1831\%$

- A 6% APR with continuous compounding results in an EAR of approximately 6.1837%.

Computing Loan Payments

Loan Payments

- Payments are made at a set interval, typically monthly.
- Each payment made includes the interest on the loan plus some part of the loan balance.
- All payments are equal and the loan is fully repaid with the final payment.

Example

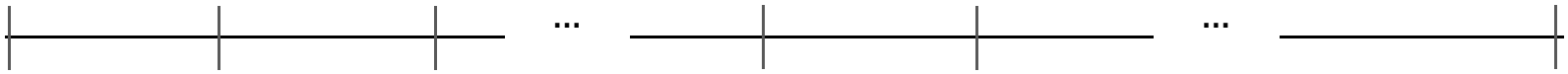
Consider a €30,000 car loan with 60 equal monthly payments, computed using a 6.75% APR with monthly compounding. What is the Loan Payment per month?

Computing the Outstanding Loan Balance

Problem

Ten years ago your firm took out a 30-year amortizing loan (€3 million) to purchase an office building. The loan has a 7.8% APR with monthly payments. How much do you owe on the loan today?

Payment per month : C?



Outstanding Loan Balance?

How are Interest Rates determined?

Fundamentally,

Interest rates are determined in the market based on **individuals' willingness to borrow and lend** (Supply and demand of financial capital)

- The **supply and demand** are in turn determined by :
  - Inflation
  - Investment and Interest Rate Policy (of the Central Bank)
  - Investment horizon
  - Risk and the identity of the borrower (i.e. the risk of default)
  - Taxes

Inflation and Interest Rates

Real *versus* Nominal Rates

Nominal Interest Rate (*r*): The rates quoted by financial institutions and used for discounting or compounding cash flows

➡ It does not represent the increase of purchasing power that will result from investing

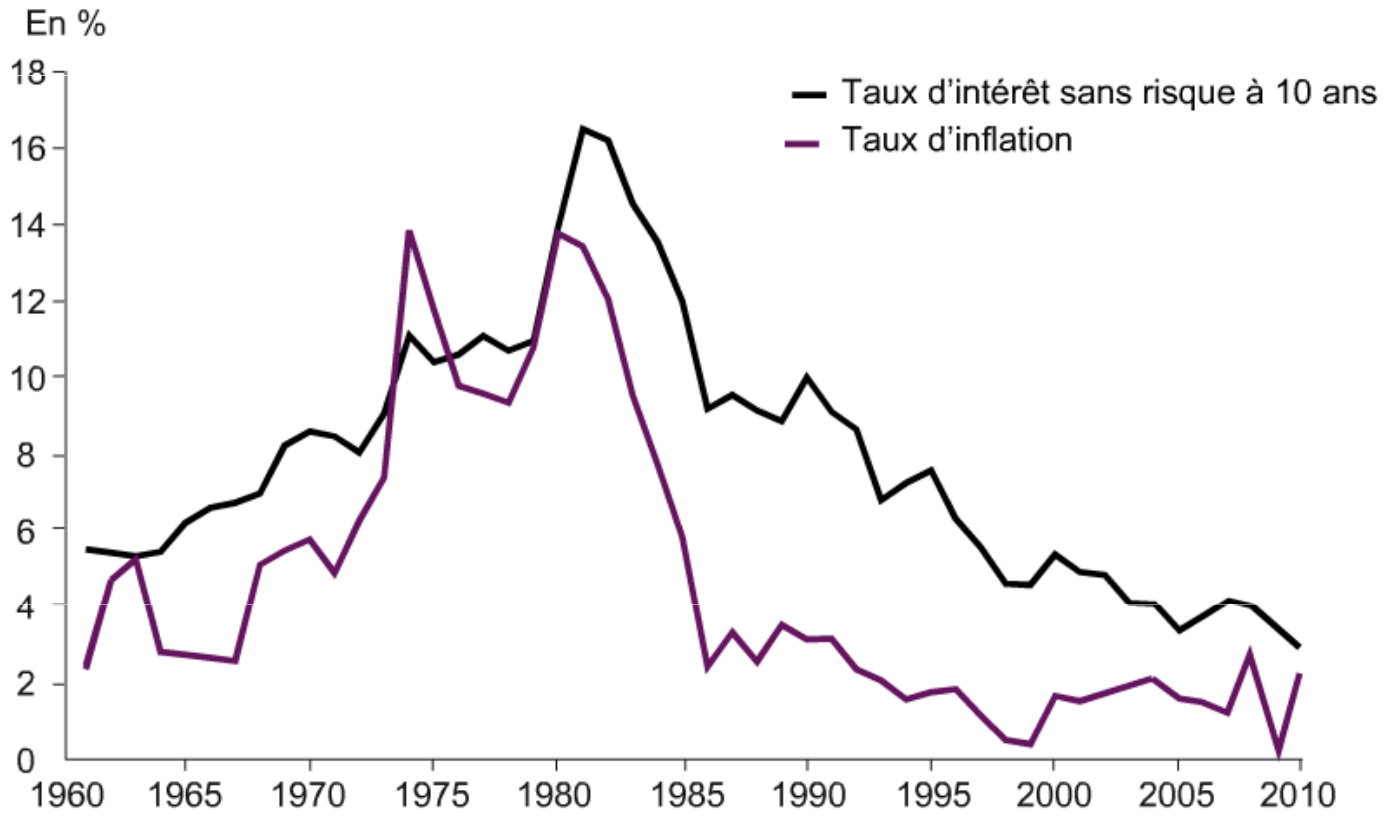
Real Interest Rate (r_r): The rate of growth of your purchasing power, after adjusting for inflation

➡ $$\text{Growth in Purchasing Power} = 1 + r_r = \frac{1+r}{1+i} = \frac{\text{Growth of Money}}{\text{Growth of Prices}}$$

Real Interest Rate: $$r_r = \frac{r-i}{1+i} \approx r - i$$

Inflation and Interest Rates

French Interest Rates and Inflation Rates 1960-2009



Source: **Berk J. and DeMarzo P. (2011)**, Finance d'entreprise, Second Edition. Pearson Education. (Fig 5.1 p.151)

Inflation and Interest Rates

Quick-check question

At the start of 2005, one-year U.S. government bond rates were about 2.8%, while the rate of inflation that year was 3.4%. At the start of 2008, one-year interest rates were about 3.2%, and inflation that year was about 0.1%. What was the real interest rate in 2005 and 2008?

Investment and Interest Rate Policy (of the Central Bank)

Interest Rate Determination

- The Central Bank (US: Federal Reserve) determines very short-term interest rates through its influence on the **reference rate** (US: federal funds rate), which is the rate at which banks can borrow cash reserves on an overnight basis.

- All other interest rates (Medium and long term rates) are set in the market and are adjusted until **the supply of lending matches the demand for borrowing** at each loan term.

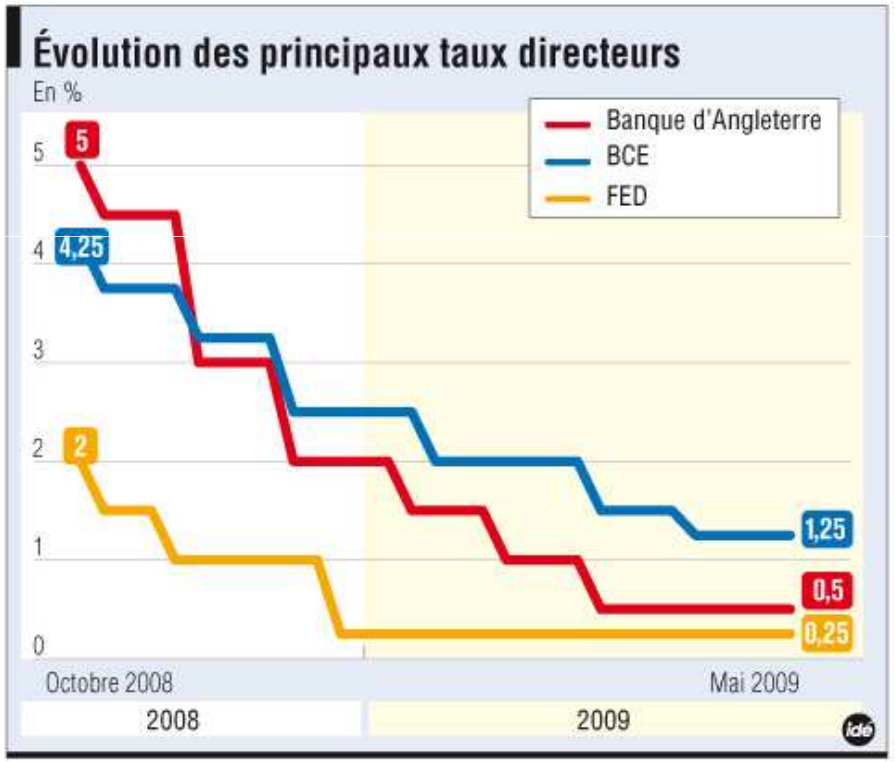
Investment and Interest Rate Policy (of the Central Bank)

Les Echos

Les Echos | 15 janvier 2009

La BCE ramène son taux directeur à 2%, Trichet ouvre la voie à une nouvelle baisse en mars

Le recul du PIB allemand annoncé hier signale une détérioration rapide de la conjoncture européenne et l'inflation est en recul. Cela a incité le conseil des gouverneurs à baisser les taux aujourd'hui plutôt que le mois prochain.



- What is the impact of an increase in interest rates on the number of investment opportunities with positive NPV?

- The Central Bank lowers interest rates to ……. investment and economic growth (and raises it to moderate investment and combat inflation)

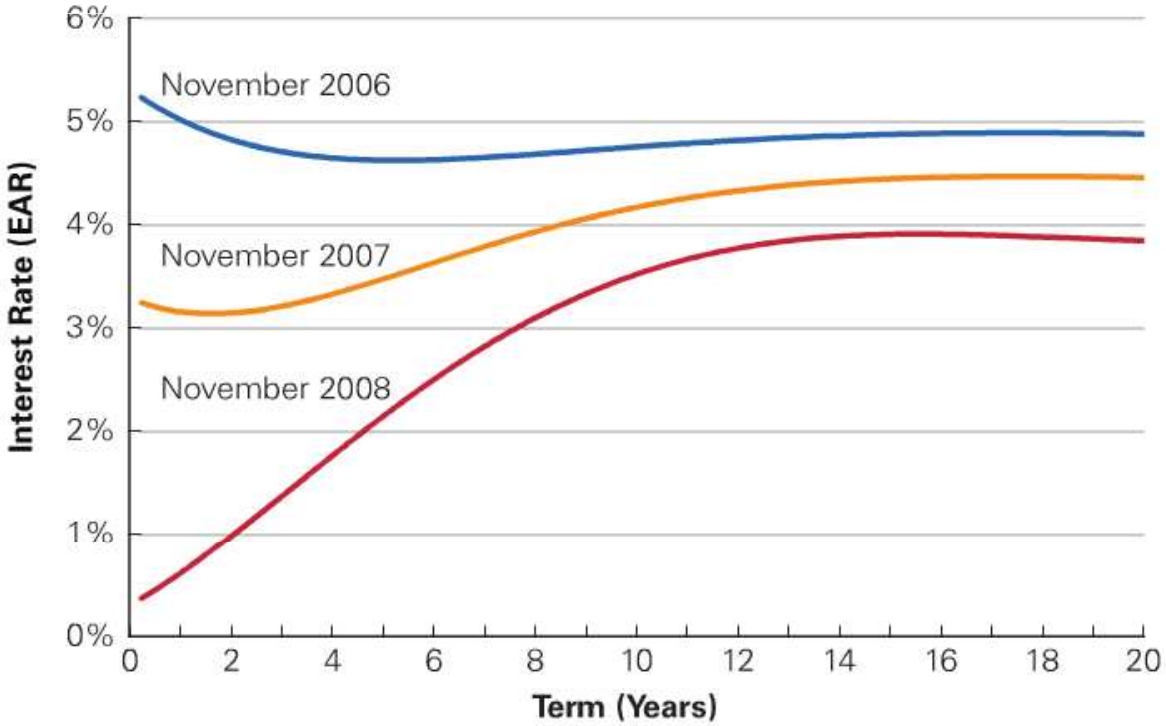
Investment horizon and Interest Rate: the Yield Curve

Term Structure: The relationship between the investment term and the interest rate

➡ **Yield Curve:** A graph of the term structure

Term (years)	Date Nov-06	Nov-07	Nov-08
0.5	5.15%	3.20%	0.44%
1	5.02%	3.15%	0.60%
2	4.83%	3.14%	0.96%
3	4.71%	3.20%	1.35%
4	4.64%	3.32%	1.75%
5	4.62%	3.47%	2.13%
6	4.62%	3.63%	2.49%
7	4.65%	3.78%	2.81%
8	4.68%	3.93%	3.09%
9	4.71%	4.06%	3.32%
10	4.75%	4.17%	3.51%
15	4.87%	4.44%	3.90%
20	4.88%	4.45%	3.84%

Term Structure of Risk-Free U.S. Interest Rates, November 2006, 2007, and 2008



Source : **Berk J. and DeMarzo P. (2011)**, Corporate Finance, Second Edition. Pearson Education. (Fig 5.2 p.138)

➡ When the 2008 financial crisis struck, the Federal Reserve responded by cutting its short-term interest rate target to 0%.

Investment horizon and Interest Rate: the Yield Curve

Interest Rate Expectations

The shape of the yield curve is influenced by interest rate **expectations**.

An inverted yield curve indicates that interest rates are expected to decline in the future

- Because interest rates tend to fall in response to an economic slowdown, an inverted yield curve is often interpreted as a negative forecast for economic growth.

Each of the last six recessions in the United States was preceded by a period in which the yield curve was inverted.

The yield curve tends to be sharply increasing as the economy comes out of a recession and interest rates are expected to rise.

Using the Term Structure to compute Present Values

The term structure can be used to compute the present and future values of a risk-free cash flow over different investment horizons.

Present Value of a Cash Flow Stream Using a Term Structure of Discount Rates

$$PV = \frac{C_1}{1 + r_1} + \frac{C_2}{(1 + r_2)^2} + \cdots + \frac{C_N}{(1 + r_N)^N} = \sum_{n=1}^{N} \frac{C_N}{(1 + r_n)^n}$$

Example

Compute the present value of a risk-free five-year annuity of $1,000 per year, given the yield curve for November 2008.

Term (years)	Date Nov-06	Nov-07	Nov-08
0.5	5.15%	3.20%	0.44%
1	5.02%	3.15%	0.60%
2	4.83%	3.14%	0.96%
3	4.71%	3.20%	1.35%
4	4.64%	3.32%	1.75%
5	4.62%	3.47%	2.13%

$PV(Annuity \text{ of } \$1000 \text{ for } 5 \text{ years}) =$

Note that the discount rates differ for each cash flow (then we cannot use the annuity formula)

Interest rates, Risk and the Identity of the Borrower (i.e. the risk of default)

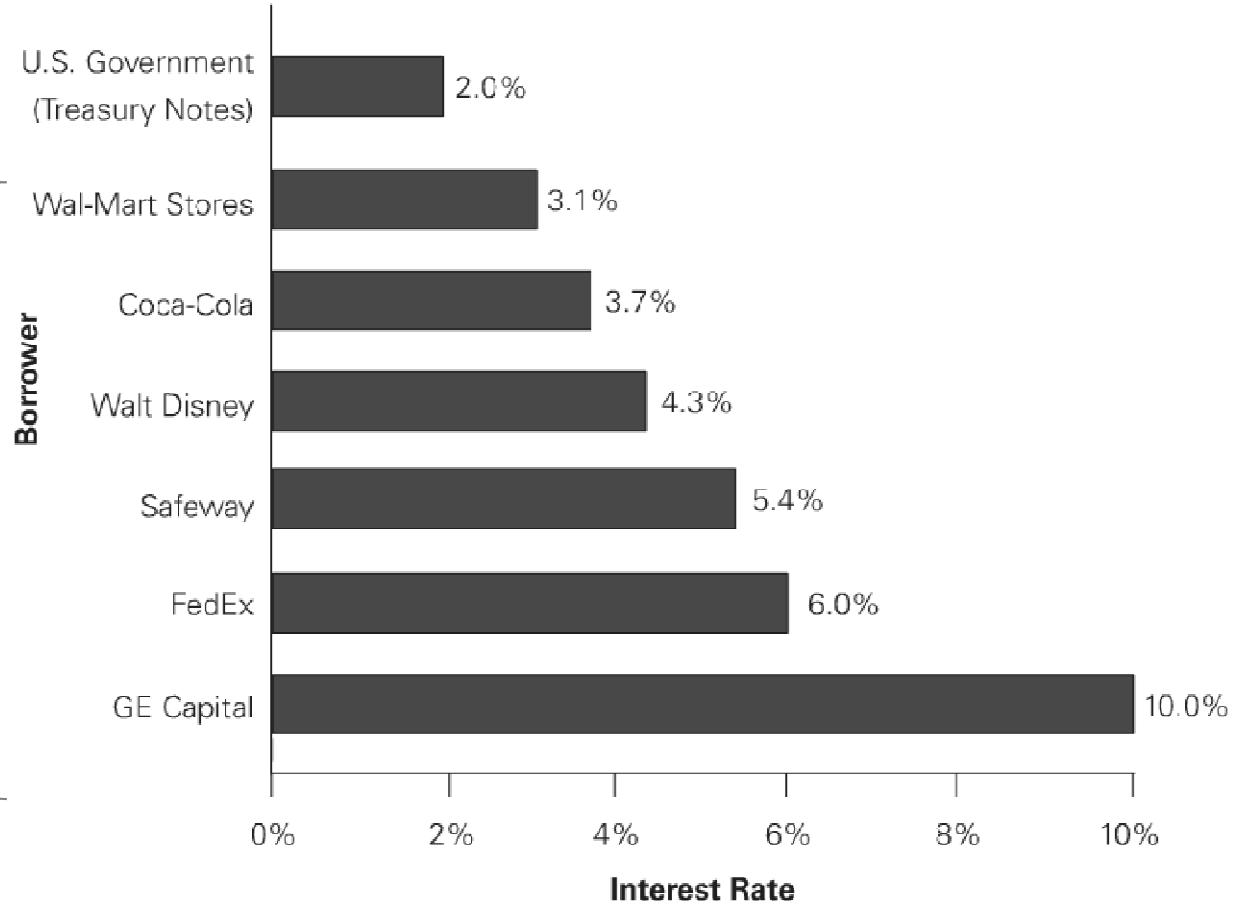
Interest Rates on Five-Year Loans (Bonds) for Various Borrowers, March 2009

Widely regarded to be "risk-free"

Risk of default ≠ 0

To compensate for the risk, investors demand a higher interest rate than r_f

=> .....



Source : **Berk J. and DeMarzo P. (2011)**, Corporate Finance, Second Edition. Pearson Education. (Fig 5.4 p.143)

➡ The variation in interest rates is based on the riskiness of the borrower (its credit standing)

The right interest-discount rate must account for the term and the risk

The Bottom line

We observe so many interest rates in the market.

Interest rates will vary based mainly on :

- Quoting conventions
- The term of investment
- The risk

➡ Which interest rate should investors/companies use as a discount rate to evaluate an investment opportunity?

➡ When discounting future cash flows, it is important to use a discount rate that matches the **horizon** and the **risk** of the cash flows

The appropriate discount rate is given by the Opportunity Cost of Capital

If an investor faced many investment choices, the opportunity cost of a given choice would be estimated by the **best available expected return offered in the market** on an investment of comparable risk and term

➡ The Opportunity Cost of Capital can be interpreted as the **return the investor forgoes** on an alternative investment of equivalent risk and term when he takes on a new investment

Basic Example

Suppose a friend offers to borrow €1,000 from you today and in return pay you €1,100 one year from today. Looking in the market for other options for investing the €1,000, you find your best alternative option that you view as equally risky as lending it to your friend. That option has an expected return of 8%.

What should you do?

The appropriate discount rate is given by the Opportunity Cost of Capital

Opportunity Cost of Capital: The best available expected return offered in the market on an investment of comparable risk and term to the cash flow being discounted (Also referred to as *Cost of Capital)*

- For a risk-free project (unlikely): OCC = interest rate on Treasury securities with a similar term
- For a risky project: the OCC will often exceed the risk-free rate, depending on the nature and magnitude of the risk

DIG DEEPER

We will develop tools for estimating the cost of capital for risky projects in the second part of this course

The appropriate discount rate is given by the Opportunity Cost of Capital

Quick Check Problem

We are at the end of the 1990s. You are the financial manager of Porsche. The company's CEO asked you to examine an investment opportunity of manufacturing a new luxury crossover (The Porsche Cayenne). What should you do ?