Financial Economics

Risk, Return and Diversification

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HEC, Paris
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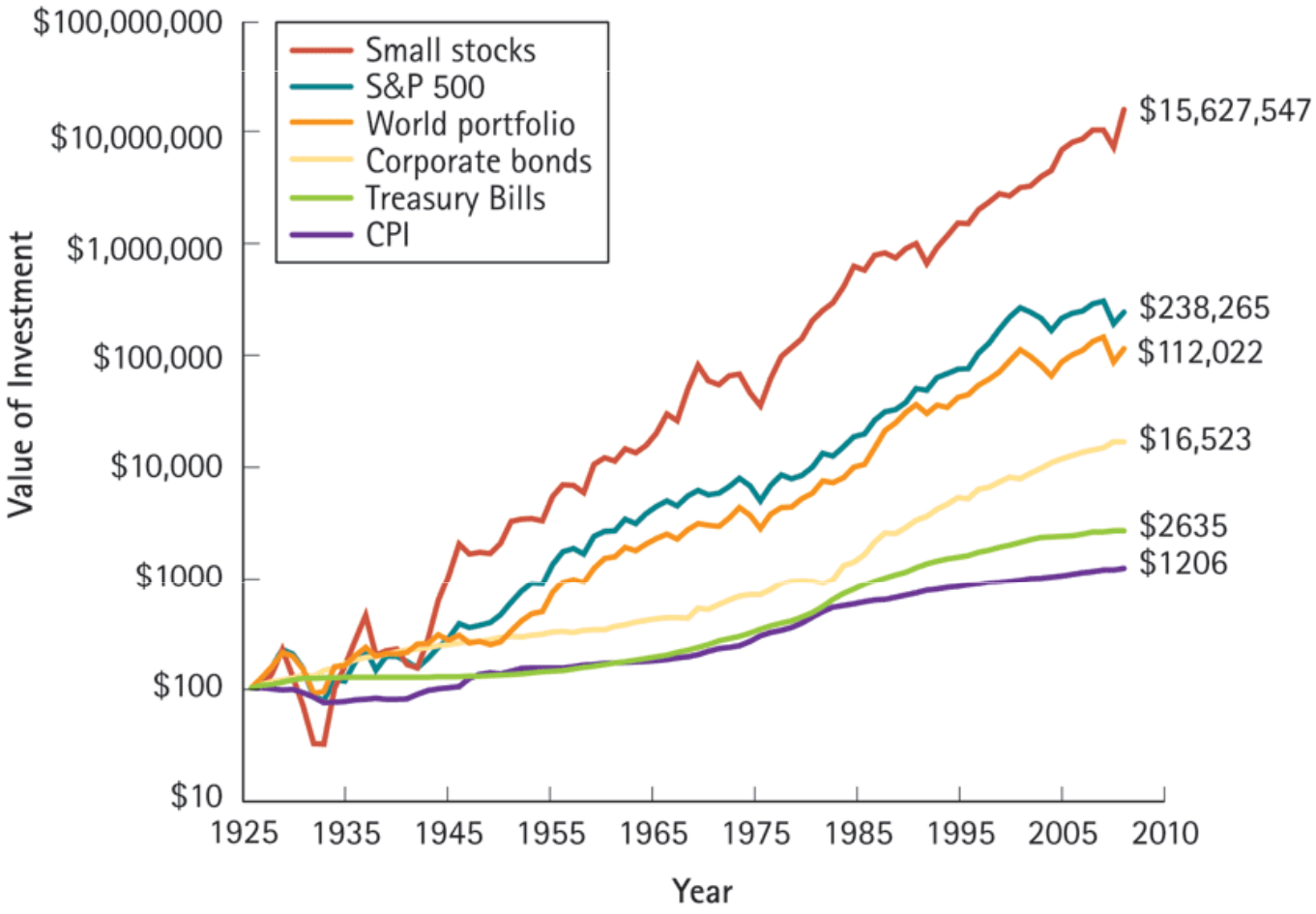
Student version

Introduction

Value of $100 Invested at the End of 1925 in U.S. Large Stocks (S&P 500), Small Stocks, World Stocks, Corporate Bonds, and Treasury Bills



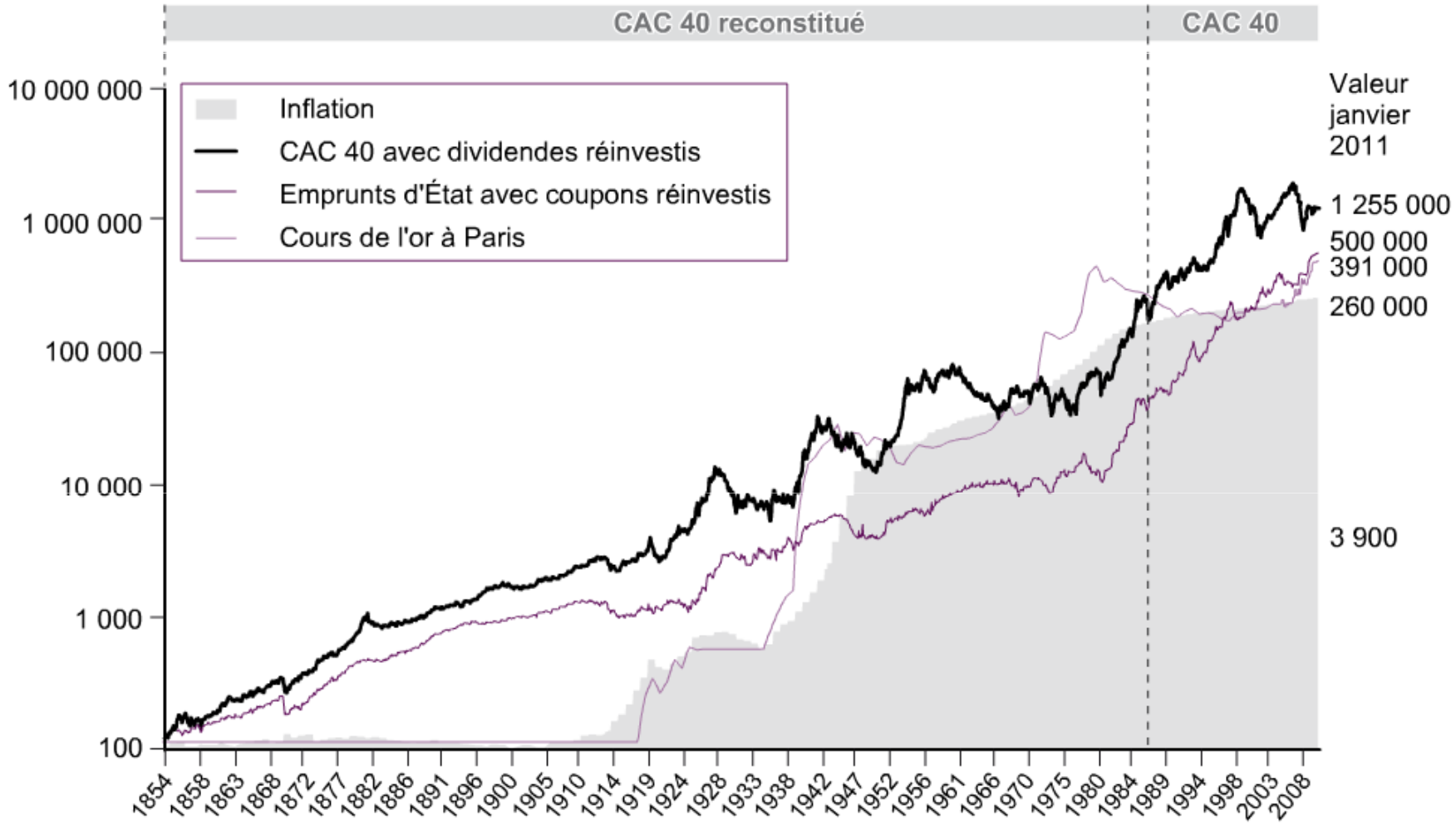
Source : Berk J. and DeMarzo P. (2011), Corporate Finance, Second Edition. Pearson Education. (Figure 10.1 p.293)

Introduction

- In the long run, small stocks experienced the highest long-term return
- Nevertheless, the value of this portfolio also experienced the largest fluctuations and then most variable returns (ex. Investors in this portfolio had the largest loss during the Depression ear of the 1930s)
- Investors in the Treasury Bills portfolio experienced any losses during the period, but enjoyed steady – albeit modest – gains each year
  - **Statistically, there is a positive <u>association</u> between risk and return**
- How much investors demand (in terms of higher expected return) to bear a given level of risk?
  - To quantify the relationship, we must first develop tools that will allow us to measure risk and return

Introduction

Value of $100 Invested at the End of 1854 in French Large stocks (CAC40), Treasury bonds, and Gold.



Source : Data provided by David Le Bris

Learning Objectives

- Identify which types of securities have historically had the highest returns and which have been the most volatile
- Compute the average return and volatility of returns from a set of historical asset prices
- Understand the tradeoff between risk and return for large portfolios versus individual stocks
- Describe the difference between common and independent risk
- Explain how diversified portfolios remove independent risk, leaving common risk as the only risk requiring a risk premium
- Calculate the expected return and volatility (standard deviation) of a portfolio
- Understand how does the correlation between the stocks in a portfolio affects the portfolio's volatility

Chapter outline

Common Measures of Risk and Return	Expected Return Historical or Realized Return Variance and Standard Deviation: Common Measures of Risk Limitations of Expected Return Estimates
The Trade-Off Risk –Return and Diversification	The Price of Risk: Risk Aversion and Risk Premium Returns of Large Portfolios Versus Returns of Individual Stocks Specific Risk Versus Systematic Risk Risk and Diversification
Measuring Return and Volatility of a Stock Portfolio	The Return of a Portfolio Combining risks: Covariance and Correlation Computing Portfolio's Volatility The bottom line

Expected Return from Probability Distributions

Probability Distributions

When an investment is risky, there are different returns it may earn. Each possible return has some likelihood of occurring. This information is summarized with a probability distribution, which assigns a probability, P_R , that each possible return, R , will occur.

Expected Return

Calculated as a weighted average of the possible returns, where the weights correspond to the probabilities.

$$\text{Expected Return} = E[R] = \sum_R P_R * R$$

Quick Check Problem

Assume BFI stock currently trades for $100 per share. In one year, there is a 25% chance the share price will be $140, a 50% chance it will be $110, and a 25% chance it will be $80. Calculate the expected return of BFI.

		Probability Distribution	
Current Stock Price ($)	Stock Price in One Year ($)	Return, R	Probability, P_R
	140	0.40	25%
100	110	0.10	50%
	80	–0.20	25%

➡ $E[R_{BFI}] =$

Computing Historical Return or Realized Return

Realized Return

The return that actually occurs over a particular time period.

$$R_{t+1} = \frac{P_{t+1} + Div_{t+1} - P_t}{P_t} = \frac{P_{t+1} - P_t}{P_t} + \frac{Div_{t+1}}{P_t}$$

$$= \text{Capital Gain Rate} + \text{Dividend Yield}$$

Quick Check Problem

Microsoft paid a one-time special dividend of $3.08 on November 15, 2004. Suppose you bought Microsoft stock for $28.08 on November 1, 2004 and sold it immediately after the dividend was paid for $27.39. What was your realized return from holding the stock?

- $R_{t+1}(\text{Microsoft}) =$

Average Annual Return

Average Annual Return

The AAR of an investment during some historical period is the average of realized returns for each year

$$\overline{R} = \frac{1}{T}(R_1 + R_2 + \ldots + R_T) = \frac{1}{T}\sum_{t=1}^{T} R_t$$

Realized return for the CAC40, Total and French Treasury Bonds (3 months)

Fin d'année	Indice CAC 40	Dividendes versés*	Rentabilité effective du CAC 40	Rentabilité effective de Total	Rentabilité moyenne annuelle des bons du Trésor à 3 mois
2000	5 926,42				
2001	4 624,58	70,71	–20,8 %	3,4 %	4,3 %
2002	3 063,91	91,00	–31,8 %	–13,0 %	3,3 %
2003	3 557,90	78,23	18,7 %	12,2 %	2,3 %
2004	3 821,16	92,10	10,0 %	14,0 %	2,0 %
2005	4 715,23	94,21	25,9 %	36,1 %	2,1 %
2006	5 541,76	130,25	20,3 %	7,8 %	2,9 %
2007	5 614,08	165,00	4,3 %	7,8 %	3,9 %
2008	3 217,97	184,46	–39,4 %	–28,3 %	3,6 %
2009	3 936,33	143,87	26,8 %	22,1 %	0,6 %
2010	3 804,78	140,61	0,2 %	–6,6 %	0,4 %

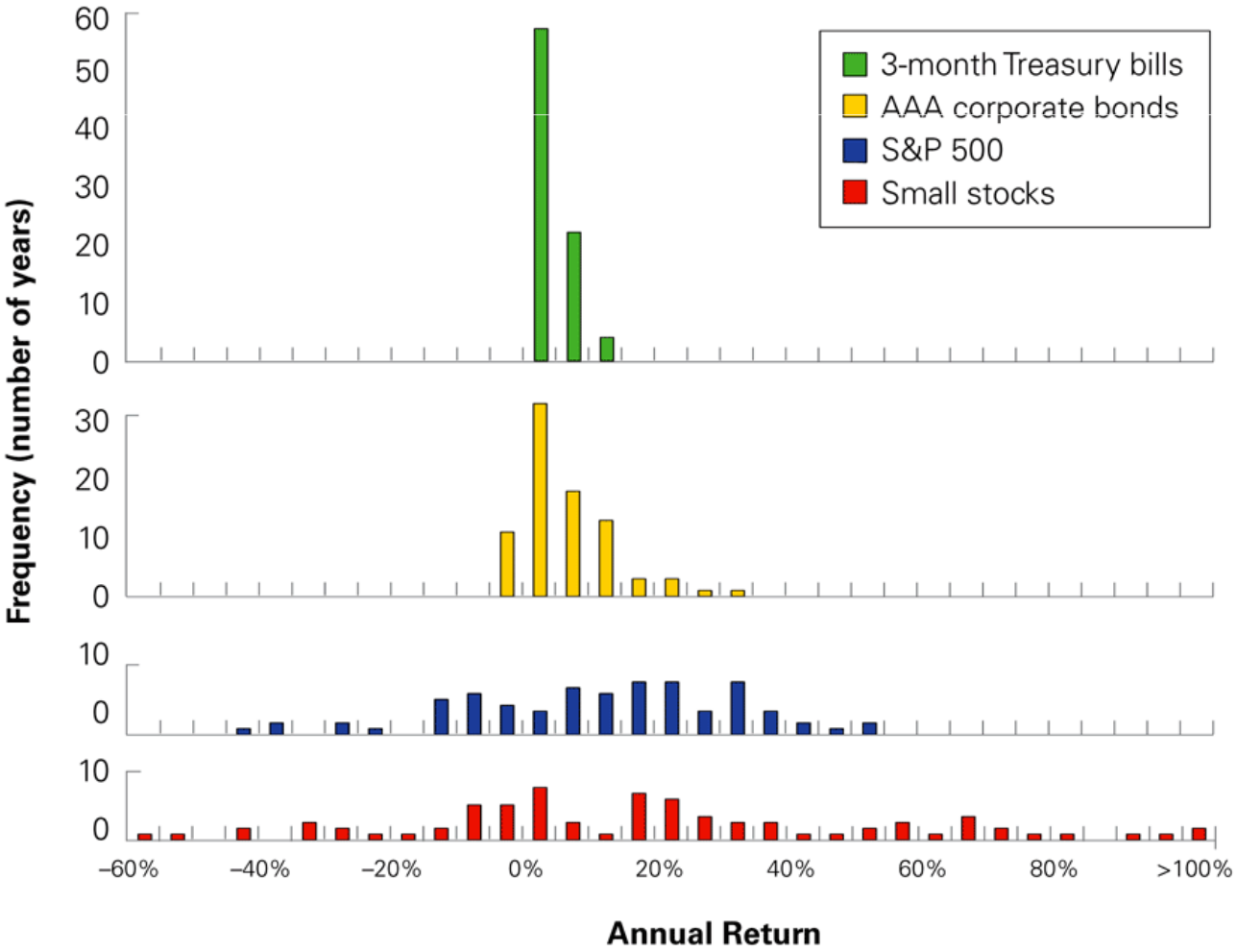
* Dividendes totaux versés par les entreprises composant l'indice CAC 40, calculés en fonction du nombre d'actions cotées sur Euronext pour chacune de ces 40 entreprises et ajustés sur 12 mois glissants en supposant qu'ils sont réinvestis dans l'indice au moment du versement.

Source : Berk J. and DeMarzo P. (2011), Finance d'entreprise, 2ème Edition. Pearson Education. (Table 10.2 p.320)

The average annual return for the CAC40 for the years 2001-2010 is:

Plotting the Historical Annual Returns in a Chart

The Empirical Distribution of Annual Returns for U.S. Large Stocks (S&P 500), Small Stocks, Corporate Bonds, and Treasury Bills, 1926–2008

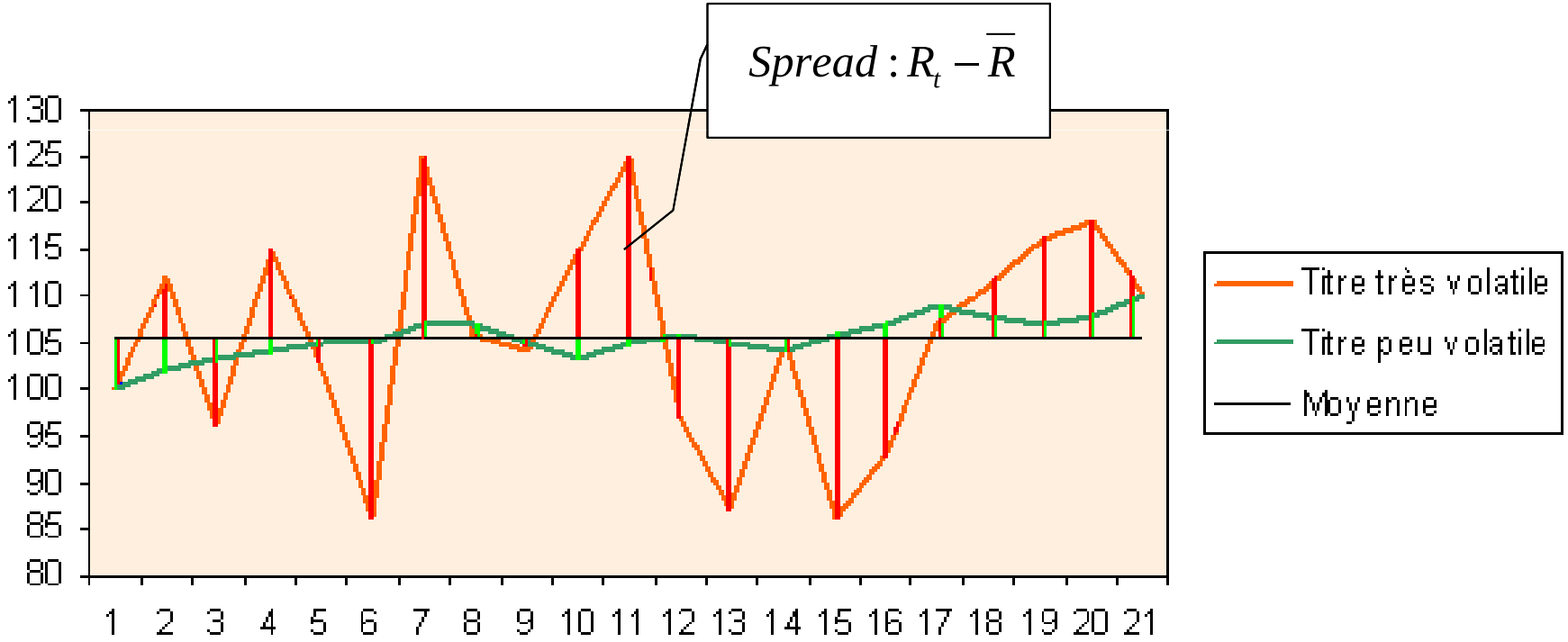


The height of each bar represents the number of years that the annual returns were in each 5% range.

Source : Berk J. and DeMarzo P. (2011), Corporate Finance, Second Edition. Pearson Education. (Figure 10.4 p. 301)

Variance and Standard Deviation are Common Measures of Risk

The risk of a security is measured by its volatility: the magnitude of the deviations from the mean



- The Standard Deviation indicates the degree of fluctuations of a security
  - The value of the orange security shows more fluctuations than the value of the green security. Its Standard Deviation is sharply higher than the green one.

Variance and Standard Deviation are Common Measures of Risk

Expected Variance

The expected squared deviation from the mean

$$Var(R) = E\left[\left(R - E[R]\right)^2\right] = \sum_R P_R \times \left(R - E[R]\right)^2$$

➡ Variance estimate Using **Expected Returns**

Historical Variance

The average squared deviation from the mean

$$Var(R) = \frac{1}{T-1}\sum_{t=1}^{T}\left(R_t - \bar{R}\right)^2$$

Standard Deviation (in Finance, Volatility)

The square root of the Variance: $SD = \sqrt{Var(R)}$

Variance and Standard Deviation are Common Measures of Risk

The bottom line: What use is Variance ?

- The variance is a measure of how « spread out » the distribution of the return is: the level of variability of the security returns

- If the return is risk-free and never deviates from its mean, the variance is equal Zero

- The variance increases with the magnitude of the deviations from the mean

  - The higher the Variance the higher the risk

Standard Deviation of a security → **Volatility**

Variance and Standard Deviation are Common Measures of Risk

Realized return for the CAC40, Total and French Treasury Bonds (3 months)

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Source : Berk J. and DeMarzo P. (2011), Finance d'entreprise, 2ème Edition. Pearson Education. (Table 10.2 p.320)

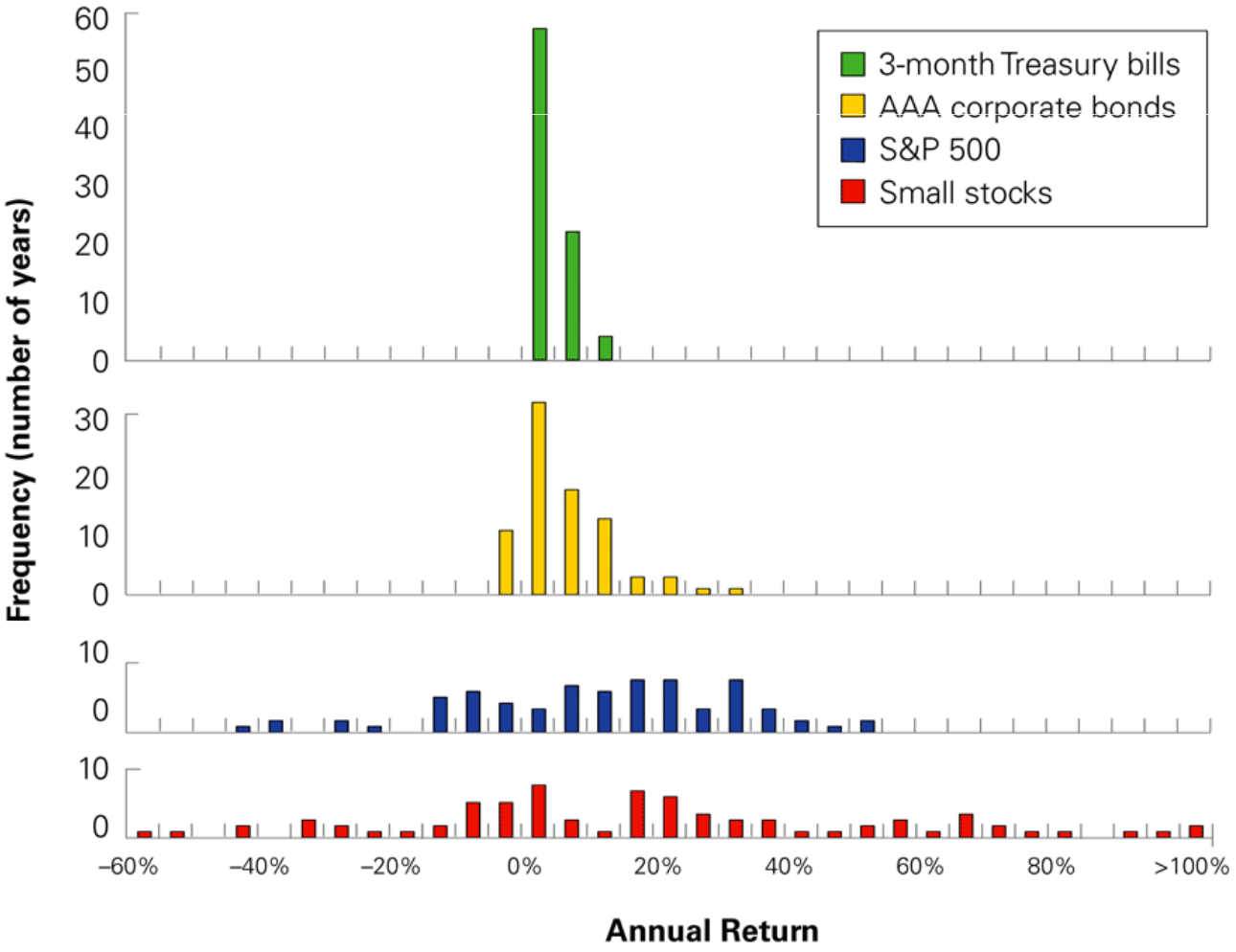
The realized Variance of the CAC40's returns for the years 2001-2010 is:

$$Var\,[R] = \frac{1}{T-1}\sum_{t=1}^{T}(R_t - \bar{R})^2$$

The historical Volatility of the CAC40 (2001-2010) is:

Variance and Standard Deviation are Common Measures of Risk

The Empirical Distribution of Annual Returns for U.S. Large Stocks (S&P 500), Small Stocks, Corporate Bonds, and Treasury Bills, 1926–2008



Historical Standard Deviation :
return volatility

3.18%

7.17%

20.36%

42.75%

Source : Berk J. and DeMarzo P. (2011), Corporate Finance, Second Edition. Pearson Education. (Figure 10.4 p. 301)

Using Past Returns to Predict the Future: Estimation Errors

We can use a security's historical average return to estimate its actual expected return, However there are many limitations to this approach

- We do not know what investors expected in the past; we can only observe realized returns

  Ex. In 2008, investors lost 37% in investing in the S&P500, which is surely not what they expected at the beginning of the year

- The average return **is just an estimate** of the expected return, and is subject to estimation errors

DIG DEEPER

More details on limitations of this approach
http://fahmi.ba.free.fr/docs/Courses/ff1_chap7.pdf

- **The average return investor earned in the past is not a reliable estimate of a security's expected return**
  - We need to derive a different method : see next chapter (CAPM)

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Risk Aversion: A bird in the hand is worth two in the bush

Cash flows and Market Prices of a Risk-Free Bond and an investment in the Market Portfolio

Security	Market Price Today	Cash Flow in one year		**Average Expected Price**
		Weak Economy P=50%	Strong Economy 1-P=50%	
Risk-free bond	1058€	1100€	1100€	**1 100€**
Market portfolio (index)	1000€	800€	1400€	**1 100€**

The market portfolio has an average expected price of: =

- Although this *average* payoff is the same as the risk-free bond, the market portfolio has a lower price today. What account for this lower price?

In general, investors don't like risk

- They prefer to have a relatively <u>safe</u> income rather than a bigger but <u>risky</u> one: **risk aversion**
- *The personal cost of losing a dollar in bad times (dissatisfaction) is greater than the benefit of an extra dollar in good times (satisfaction)*

In finance, we assume that investors are risk averse

How risk aversion impact investment decisions?

The more risk averse investors are, the …… the current price of the risky security will be compared to a risk-free bond with the same average payoff

When investing in risky project, investors will expect a return that appropriately compensates them for the risk

- **Risk Premium**

  Additional return that investors expect to earn to compensate them for the security's risk

Estimating the Risk Premium

Cash flows and Market Prices of a Risk-Free Bond and an investment in the Market Portfolio

Security	Market Price Today	Cash Flow in one year		**Average Expected Price**	**Expected Return rate E[R]**
		Weak Economy P=50%	Strong Economy 1-P=50%		
Risk-free bond	1058€	1100€	1100€	**1 100€**	**4%**
Market portfolio (index)	1000€	800€	1400€	**1 100€**	**10%**

In order to estimate the Risk Premium, we should:

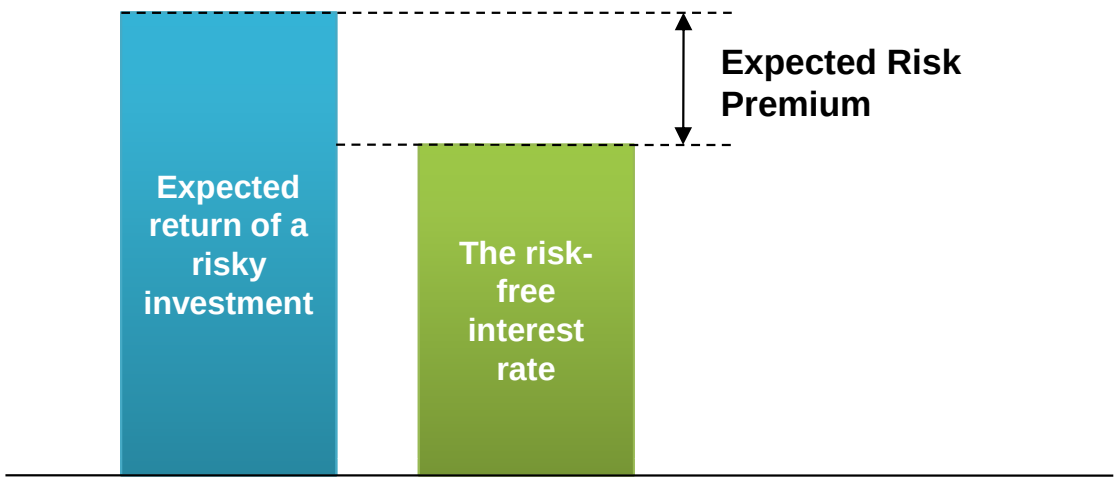
- **Calculate the difference between the expected return of the risky investment and the risk-free interest rate**

- **Risk Premium =**

Risk Premium: The bottom line

Risk Premium

Additional return (compared to risk-free interest rate) that investors expect to earn to compensate them for the security's risk



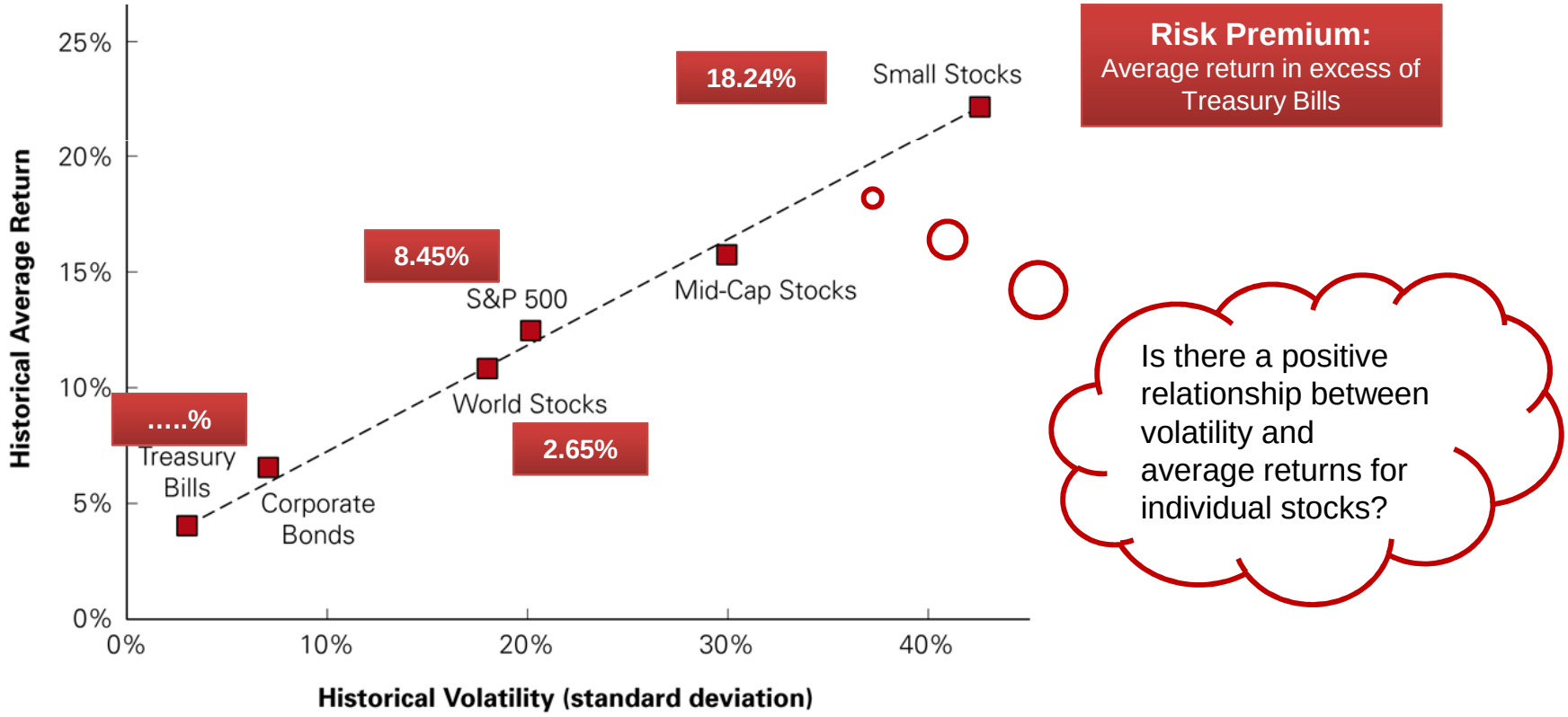
$$E[R_s] = r_f + (\text{Risk Premium of } s)$$

$E[R_s]$: Expected Return of a risky investment s

r_f : The risk - free interest rate

The returns of large portfolios

The Historical Tradeoff Between Risk and Return in Large Portfolios, 1926–2005

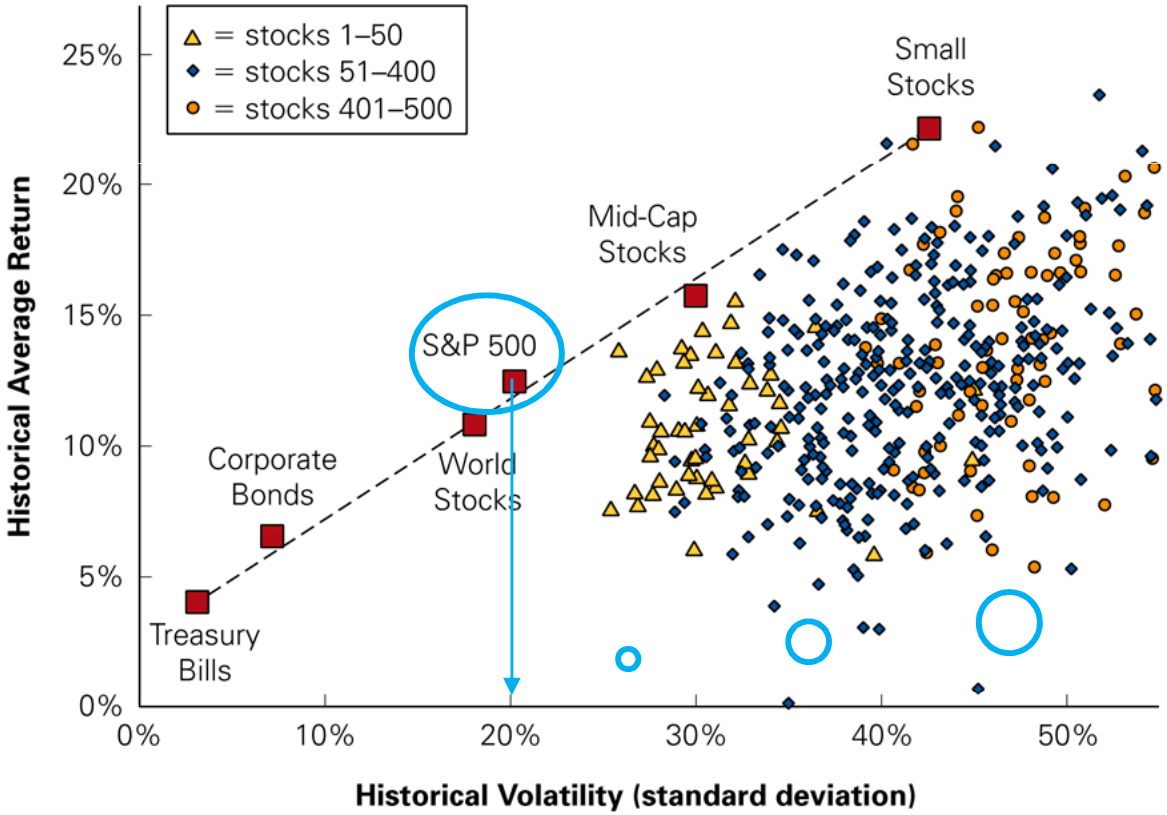


Source : Berk J. and DeMarzo P. (2011), Corporate Finance, Second Edition. Pearson Education. (Figure 10.5 p. 306)

➡ **The investments with higher volatility have rewarded investors with higher average returns**

The returns of Individual Stocks

Historical Volatility and Return for 500 Individual Stocks, by Size, Updated Quarterly, 1926–2005



How is that the S&P500 is so much less risky than all of the 500 stocks individually?

Source : Berk J. and DeMarzo P. (2011), Corporate Finance, Second Edition. Pearson Education. (Figure 10.6 p. 307)

There is no precise relationship between volatility and average return for individual stocks.

- Larger stocks tend to have ……. volatility than smaller stocks
- All stocks tend to have …….. risk than the S&P500 portfolio

Independent Risk Versus Common Risk

Theft Versus Earthquake Insurance : An example of insurance company in San Francisco

Consider two types of home insurance policies: the 1st covers the theft risk, the 2nd covers the earthquake risk

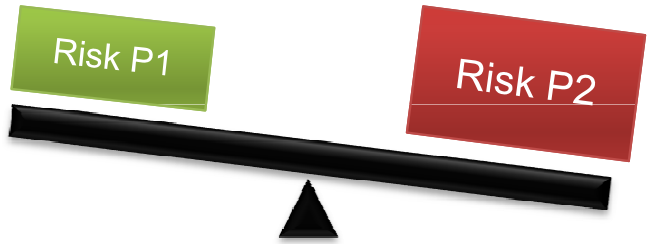
Let's assume that: **theft risk = earthquake risk = 1%** (each year there is about 1% chance that the home will be robbed and 1% chance that the home will be damaged by an earthquake). The insurance company sold 100 000 policies of each type for homeowners.

We know that the risks of the individual policies are similar (*pr =1%*), but are the risks of the portfolios of policies similar?

2 portfolios

P1 : Portfolio of policies / **theft risk**

P2 : Portfolio of policies / **earthquake risk**



P1 : **Independant risks**
The risk of theft is uncorrelated and independant across homes

P2 : **Commun Risk**
An earthquake affects all houses simultaneously: the risk is correlated across homes

➡ The P1 is less risky because it includes securities with independent risks: the averaging out of independent risks in a portfolio is called **diversification**

Independent Risk Versus Common Risk

Common Risk

Risk that is perfectly correlated

Risk that affects all securities

Independent Risk

Risk that is uncorrelated

Risk that affects a particular security

Diversification

The averaging out of independent risks in a large portfolio

- **The risk of a portfolio depends on whether the individual risks within it are common or independent**
- **What are implications of this distinction for the risk of stock portfolios?**

Firm-Specific Versus Systematic Risk

What causes stock prices to be higher or lower than we expect?

15/03/11 | | Thibaut Madelin

Areva dévisse en Bourse

Le groupe ne veut pas croire dans un nouvel hiver nucléaire après l'accident survenu au Japon, pays dans lequel il réalise 7 % de son chiffre d'affaires.

Firm-Specific Versus Systematic Risk

Usually, stock prices fluctuate due to <u>two types of news</u>

Firm-Specific News	Market-Wide News
Good or bad news about the company itself - A firm might announce a new contract which will potentially boost its sales - An unexpected CEO departure - Best employees hired away	News about the economy as a whole and therefor affects all stocks - The Central European Bank might announce that it will lower interest rates to boost the economy - 9/11 terrorist attaks - The 2008 Financial Crisis
⬇	⬇
Independant Risks	**Commun risks**
⬇	⬇
Firm-Specific, Idiosyncratic or Unsystematic Risk	Systematic or Market Risk
⬇	⬇
Diversifiable Risk	**Undiversifiable Risk**

Firm-Specific Versus Systematic Risk

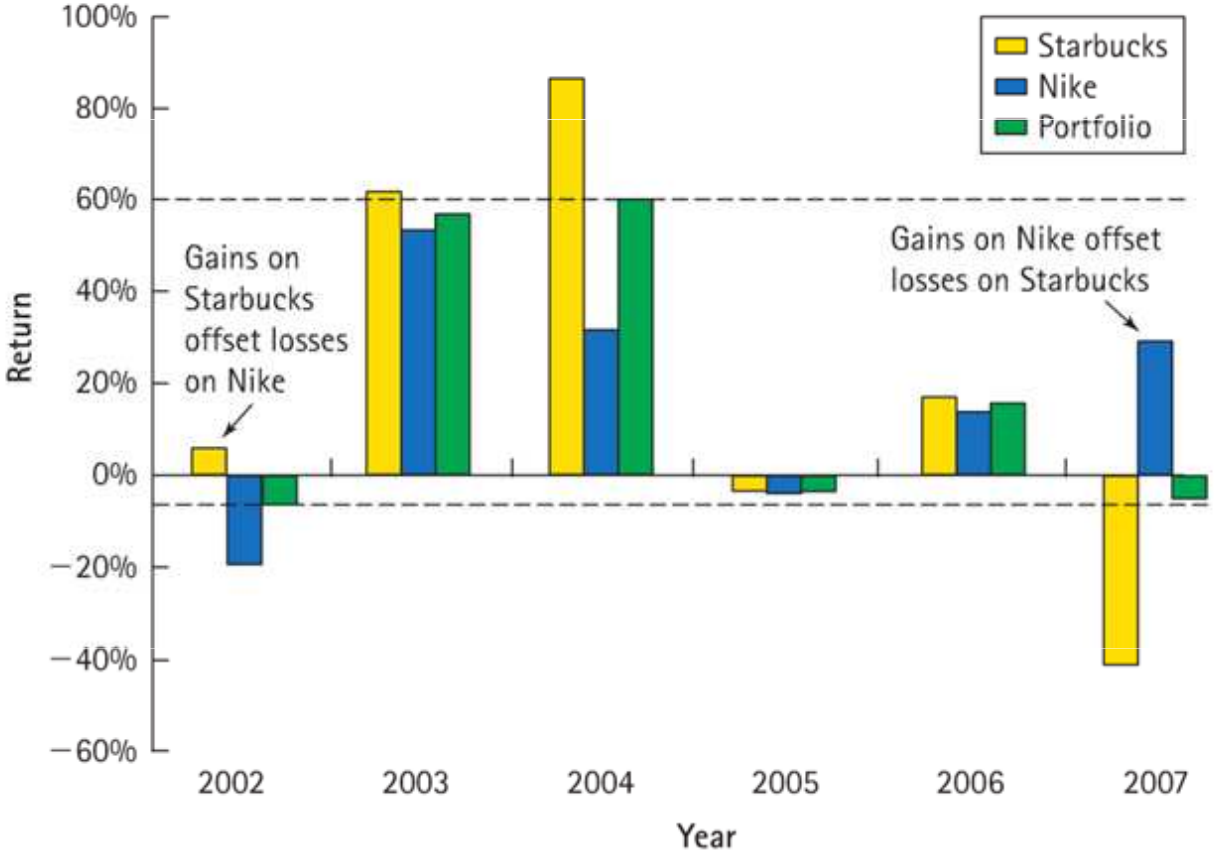
Qucik-Check Questions

Which of the following risks of a stock are likely to be firm-specific, and which are likely to be systematic risks?

1. The risk that the founder and CEO retires
2. The risk that oil prices rise, increasing production costs
3. The risk that a product design is faulty and the product must be recalled
4. The risk that the economy slows, reducing demand for the firm's products

Risk and Diversification

The effect of Diversification on Portfolio Volatility



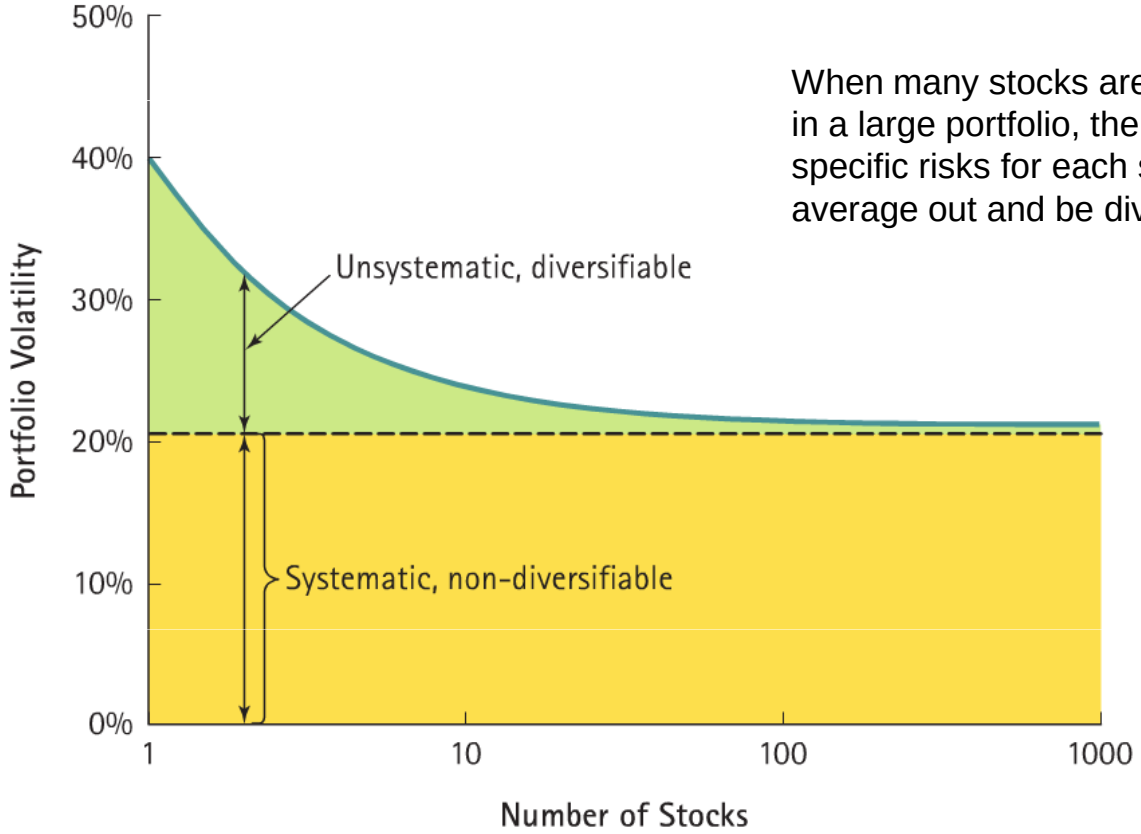
Source : Berk J. and DeMarzo P. (2012), Fundamentals of Corporate Finance. Pearson Education. (Figure 11.7 p. 336)

Portfolio's worst return is better than the worst return of either stock on its own

Risk and Diversification

The effect of Diversification on Portfolio Volatility

No. Stocks	Portfolio Volatility
1	40.0%
2	32.0%
3	28.8%
4	27.1%
5	26.0%
10	23.7%
15	22.9%
20	22.5%
25	22.2%
30	22.1%
50	21.7%
100	21.5%
1000	21.2%



When many stocks are combined in a large portfolio, the firm-specific risks for each stock will average out and be diversified

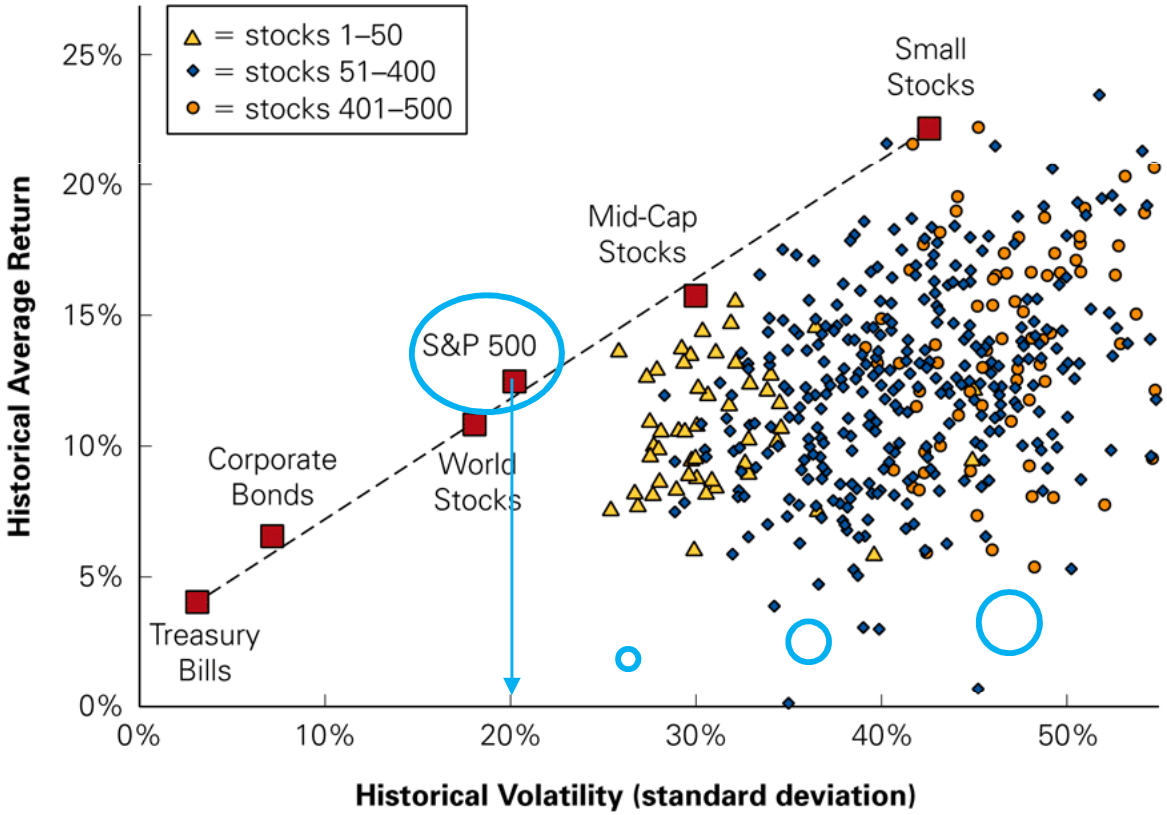
The systematic risk, however, will affect all firms and will not be diversified

Source : Berk J. and DeMarzo P. (2012), Fundamentals of Corporate Finance. Pearson Education. (Figure 12.4 p. 357)

The volatility declines with the size of the portfolio thanks to diversification of specific risks

Risk and Diversification

Historical Volatility and Return for 500 Individual Stocks, by Size, Updated Quarterly, 1926–2005



How is that the S&P500 is so much less risky than all of the 500 stocks individually?

Source : Berk J. and DeMarzo P. (2011), Corporate Finance, Second Edition. Pearson Education. (Figure 10.6 p. 307)

The individual stocks each contain firm-specific risk, which can be eliminated when we combine them into a portfolio

- The portfolio as a whole can have lower volatility than each of the stocks within it

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Calculating the Return of a Portfolio

Portfolio Weights

The fraction of the total investment in the portfolio held in each individual investment in the portfolio:

$$x_i = \frac{\text{Value of investment } i}{\text{Tota value of portfolio}} \qquad \sum_i x_i = 1 \text{ or } 100\%$$

<u>Historical</u> Return of a Portfolio

$$R_P = x_1 R_1 + x_2 R_2 + \cdots + x_n R_n = \sum_i x_i R_i$$

<u>Expected</u> Return of a Portfolio

$$E[R_P] = E\left[\sum_i x_i R_i\right] = \sum_i E[x_i R_i] = \sum_i x_i E[R_i]$$

$$\Rightarrow \quad E[R_P] = \sum_i x_i E[R_i]$$

Calculating the Return of a Portfolio

Problem : Portfolio Returns

Suppose you buy 200 shares of the BNP Company at €30 per share and 100 shares of Pernod-Ricard at €40 per share. If BNP's share price goes up to €36 and Pernod-Ricard's falls to €38. what return did the portfolio earn? After the price change, what are the new portfolio weights?

$$x_{BNP} = \frac{\text{Value of investment } i}{\text{Tota value of portfolio}} =$$

$$x_{PRi} =$$

$$R_{BNP_{t+1}} = \frac{P_{t+1} + Div_{t+1} - P_t}{P_t} = \ldots$$

$$R_{PR_{t+1}} =$$

$$R_P = x_{BNP}.R_{BNP} + x_{PR}.R_{PR} = \ldots\ldots\ldots\ldots$$

The value of the new portfolio:

$$x_{BNP} =$$

$$x_{PRi} =$$

Combining Risks

Returns for Three Stocks, and Portfolios of Pairs of Stocks

	Stock Returns			Portfolio Returns	
Year	North Air	West Air	Tex Oil	$1/2R_N + 1/2R_W$	$1/2R_W + 1/2R_T$
2003	21%	9%	–2%	15.0%	3.5%
2004	30%	21%	–5%	25.5%	8.0%
2005	7%	7%	9%	7.0%	8.0%
2006	–5%	–2%	21%	–3.5%	9.5%
2007	–2%	–5%	30%	–3.5%	12.5%
2008	9%	30%	7%	19.5%	18.5%
Average Return	10.0%	10.0%	10.0%	10.0%	10.0%
Volatility	13.4%	13.4%	13.4%	12.1%	5.1%

By combining stocks into a portfolio, we ……

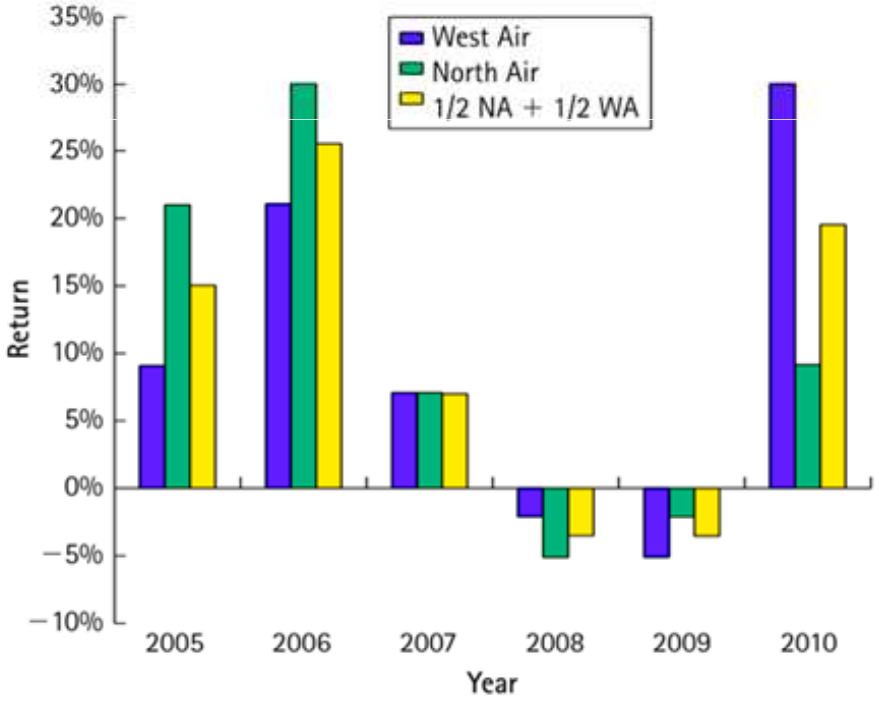
➡ Both portfolios have lower risk than the individual stocks

The amount of risk that is eliminated in a portfolio depends on the degree to which the stocks face ……and their prices move together

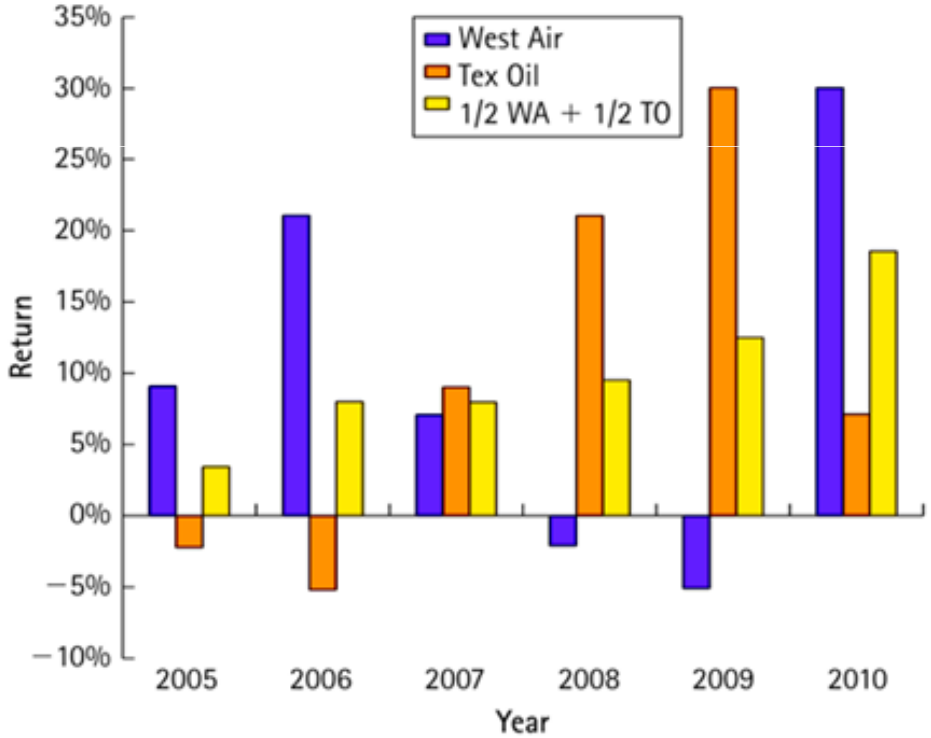
➡ To find the risk of a portfolio, one must know the degree to which the stocks' returns move together.

Combining Risks

Portfolio split equally between North Air and West Air



Portfolio split equally between West Air and Texas Oil



To find the risk of a portfolio, one must know the degree to which the stocks' returns move together.

➡ **Covariance**

Covariance: a Statistical Measure of Co-movement of Returns

Covariance

The product of the deviations of two returns from their means

Expected Covariance between Returns R_i and R_j

$$Cov(R_i, R_j) = E[(R_i - E[R_i])(R_j - E[R_j])]$$

Historical Covariance between Returns R_i and R_j

$$Cov(R_i, R_j) = \frac{1}{T-1}\sum_t (R_{i,t} - \bar{R}_i)(R_{j,t} - \bar{R}_j)$$

If $Cov(R_i, R_j) > 0$:

- The two returns tend to move together

If $Cov(R_i, R_j) < 0$:

- The two returns tend to move in opposite directions

Covariance: a Statistical Measure of Co-movement of Returns

Example : Computing Covariance

What is the covariance between North Air and West Air in 2003 and 2004?

Historical Covariance between Returns R_i and R_j

$$Cov(R_i,R_j) = \frac{1}{T-1}\sum_t (R_{i,t} - \bar{R}_i)(R_{j,t} - \bar{R}_j)$$

Dates	Deviation from the mean $(R_{NA} - \bar{R}_{NA})$	$(R_{WA} - \bar{R}_{WA})$	Cov (North Air, West Air)
2003	11%	-1%	-0.0011 1
2004	20%	11%	0.0220 2

1

2

- While the covariance indicates the sign of the variation, it gives no information about its magnitude
  - In order to quantify the strength of the relationship between them, we can calculate the **Correlation**

What does the correlation measure?

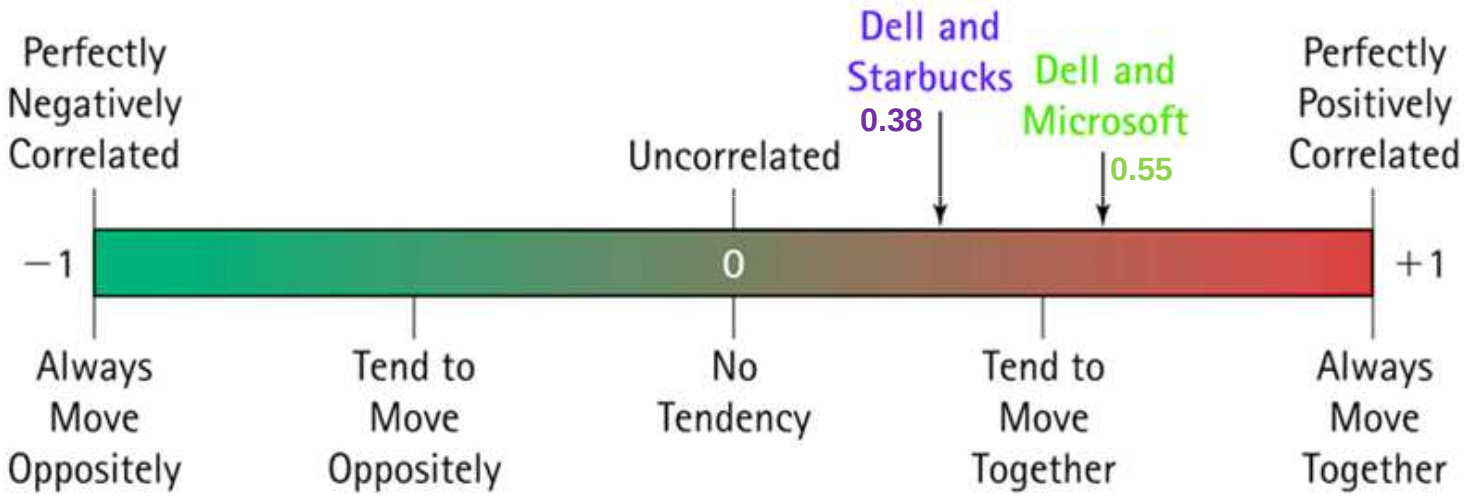
Correlation

Measure the strength of the relationship between two variables

Correlation between Returns R_i and R_j

$$Corr(R_i, R_j) = \frac{Cov(R_i, R_j)}{SD(R_i).SD(R_j)}$$

The correlation between two stocks will always be between –1 and +1



Source : Berk J. and DeMarzo P. (2012), Fundamentals of Corporate Finance. Pearson Education. (Figure 12.2 p. 352)

Correlation: a Statistical Measure of the Dependence between two variables

Example : Computing Covariance and Correlation between pairs of stocks

What is the covariance and the correlation between North Air and West Air in the period of 2003-2008?

	Deviation from Mean			North Air and West Air	West Air and Tex Oil
Year	$(R_N - \bar{R}_N)$	$(R_W - \bar{R}_W)$	$(R_T - \bar{R}_T)$	$(R_N - \bar{R}_N)(R_W - \bar{R}_W)$	$(R_W - \bar{R}_W)(R_T - \bar{R}_T)$
2003	11%	–1%	–12%	–0.0011	0.0012
2004	20%	11%	–15%	0.0220	–0.0165
2005	–3%	–3%	–1%	0.0009	0.0003
2006	–15%	–12%	11%	0.0180	–0.0132
2007	–12%	–15%	20%	0.0180	–0.0300
2008	–1%	20%	–3%	–0.0020	–0.0060
		$\text{Sum} = \sum_t (R_{i,t} - \bar{R}_i)(R_{j,t} - \bar{R}_j) =$		0.0558	–0.0642
Covariance:		$Cov(R_i, R_j) = \frac{1}{T-1}\text{Sum} =$		0.0112	–0.0128
Correlation:		$Corr(R_i, R_j) = \frac{Cov(R_i, R_j)}{SD(R_i)\,SD(R_j)} =$		0.624	–0.713

➡ The returns of NA and WA tend to move together … because of their ……

Correlation: a Statistical Measure of the Dependence between two variables

Example : Estimated Annual Volatilities and Correlations for Selected Stocks.
(Based on Monthly Returns, June 2002- May 2010)

Standard Deviation	Apple 49%	Microsoft 28%	Target 31%	Starbucks 39%	Dell 39%	HP 32%	Berkshire Hathaway 20%
Apple	1.00	0.40	0.33	0.37	0.47	0.37	0.07
Microsoft	0.40	1.00	0.36	0.36	0.55	0.52	0.32
Target	0.33	0.36	1.00	0.49	0.40	0.44	0.37
Starbucks	0.37	0.36	0.49	1.00	0.38	0.37	0.27
Dell	0.47	0.55	0.40	0.38	1.00	0.50	0.21
HP	0.37	0.52	0.44	0.37	0.50	1.00	0.33
Berkshire Hathaway	0.07	0.32	0.37	0.27	0.21	0.33	1.00

Source : Berk J. and DeMarzo P. (2012), Fundamentals of Corporate Finance. Pearson Education. (Figure 12.3 p. 353)

	Peugeot	Renault	BNP Paribas	Société Générale	Vivendi Universal
Écart type	27 %	35 %	23 %	28 %	34 %
Corrélation avec					
Peugeot	100 %	57 %	43 %	50 %	33 %
Renault	57 %	100 %	31 %	31 %	32 %
BNP Paribas	43 %	31 %	100 %	82 %	39 %
Société Générale	50 %	31 %	82 %	100 %	38 %
Vivendi Universal	33 %	32 %	39 %	38 %	100 %

Computing a Portfolio's Variance and Volatility

The Variance of a Two-Stock Portfolio

$$Var(R_P) = x_1^2 Var(R_1) + x_2^2 Var(R_2) + 2x_1 x_2 Cov(R_1, R_2)$$

$$Var(R_P) = x_1^2 Var(R_1) + x_2^2 Var(R_2) + 2x_1 x_2 Corr(R_1, R_2) SD(R_1) SD(R_2)$$

The Variance of a Large Portfolio

$$Var(R_P) = \sum_i \sum_j x_i x_j Cov(R_i, R_j)$$

The Volatility of a Portfolio

$$SD = \sqrt{Var(R_P)}$$

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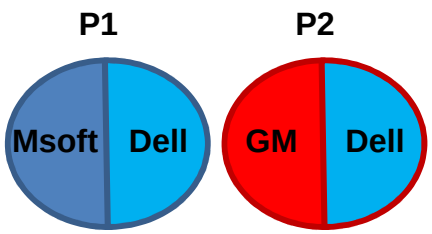
See the derivation of these formulas in the appendix

How does the correlation between the stocks in a portfolio affects the portfolio's volatility ?

Problem: Computing the Volatility of a Two-Stock portfolio

What is the volatility of a portfolio with equal amounts invested in Microsoft and Dell (P1)? Same question for General Motors and Dell (P2)?

	Microsoft	Dell	GM
Standard Deviation	37%	50%	38%
Correlation with			
Microsoft	100%	62%	25%
Dell	62%	100%	19%
GM	25%	19%	100%



The Variance of P1 and P2

The Volatility of P1 and P2:

➡

➡

How does the correlation between the stocks in a portfolio affects the portfolio's volatility ?

The total expected return of a portfolio is influenced by the return of each stock in the portfolio and their portfolio weights

The total volatility of a portfolio is influenced by the volatility of each stock in the portfolio, their portfolio weights <u>and</u> the proportion of their <u>common exposure to market risk</u>

- **The correlation between stocks in a portfolio affect its volatility, but not its expected return**
- **The lower the correlation between stocks of a portfolio, the lower is the volatility of the portfolio**
  - Lower correlation between stocks leads to greater diversification