Financial Economics

Optimal Portfolio Choice and the CAPM

Fahmi Ben Abdelkader ©
HEC, Paris
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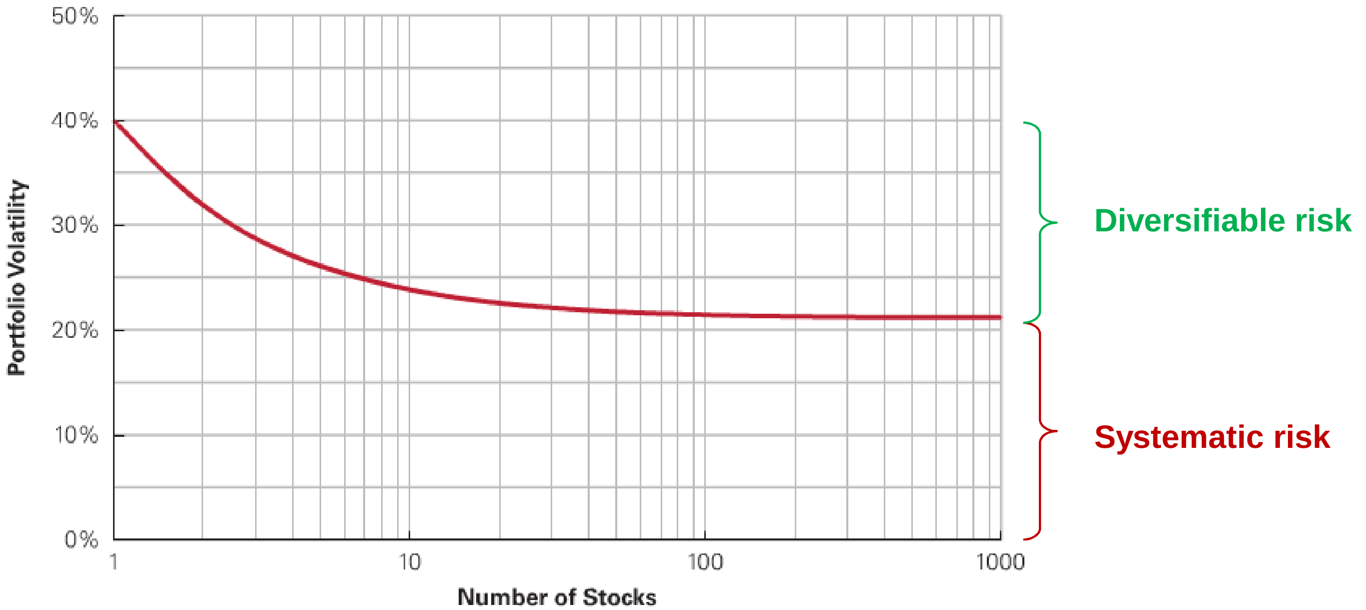
Student version

Introduction

Diversification with an Equally Weighted Portfolio of Many Stocks



Source : Berk J. and DeMarzo P. (2011), Corporate Finance, Second Edition. Pearson Education. (Figure 11.2 p.339)

The benefit of diversification declines as the number of stocks in the portfolio grows

- The decrease in volatility when going from one to two stocks is much larger than the decrease when going from 100 to 101 stocks
- To what extent should we keep adding stocks? How an investor can reach an **optimal diversification?**

Learning Objectives

- Define an efficient portfolio and a market portfolio
- Explain how an individual investor will choose from the set of efficient portfolios.
- Describe what is meant by a short sale, and illustrate how short selling extends the set of possible portfolios.
- Explain the effect of combining a risk-free asset with a portfolio of risky assets, and compute the expected return and volatility for that combination.
- Illustrate why the risk-return combinations of the risk-free investment and a risky portfolio lie on a straight line.
- Understand the relation between systematic risk and the market portfolio

Chapter Outline

Choosing an Efficient Portfolio	Efficient Portfolios Short Sales Efficient Frontier and diversification Identifying The Optimal Risky Portfolio
Measuring Systematic Risk: the Beta	Identifying Systematic Risk: The Market Portfolio The Beta: A measure of Systematic Risk Calculating and Interpreting Betas The Beta in practice
The Capital Asset Pricing Model (CAPM)	The CAPM Assumptions and the Market Portfolio Measuring the Cost of Capital: the CAPM Equation The Capital Market Line Versus The Security Market Line Summary of the CAPM

Efficient portfolios with two stocks

Example

Consider a portfolio of Intel and Coca-Cola. Suppose an investor believes these stocks are uncorrelated and will perform as follows:

	Expected Return	Volatility
Intel	26%	50%
Coca-Cola	6%	25%

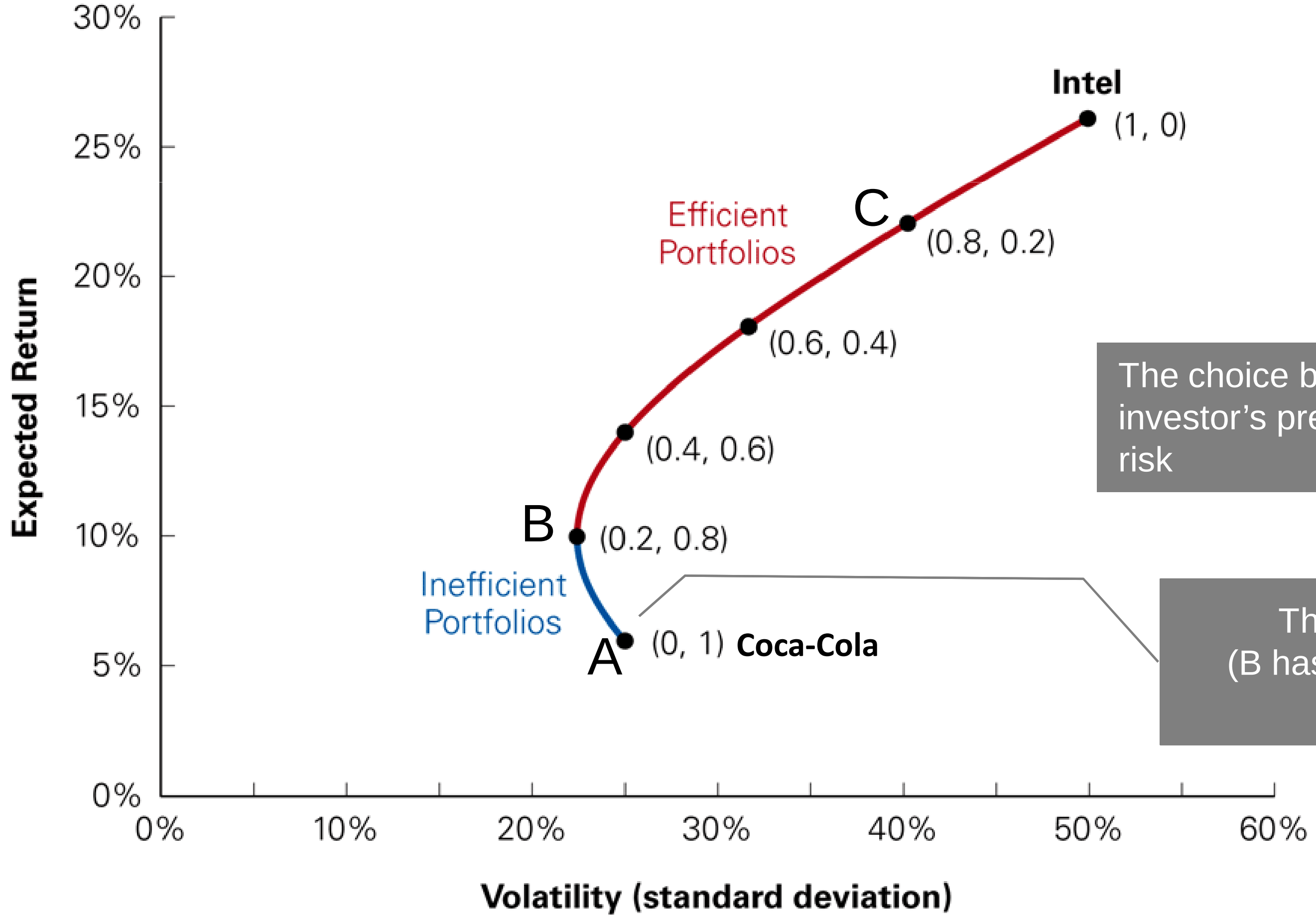
How should the investor choose a portfolio of these two stocks? Are some portfolios preferable to others?

- Let's compute the expected return and volatility for different combinations of the stocks

Portfolio Weights		Expected Return (%)	Volatility (%)
x_I	x_C	$E[R_P]$	$SD[R_P]$
0%	100%	6%	25.0%
20%	80%	10%	22.3%
40%	60%	14%	25.0%
60%	40%	18%	31.6%
80%	20%	22%	40.3%
100%	0%	26%	50.0%

Efficient portfolios with two stocks

Volatility Versus Expected Return for Portfolios of Intel and Coca-Cola Stock



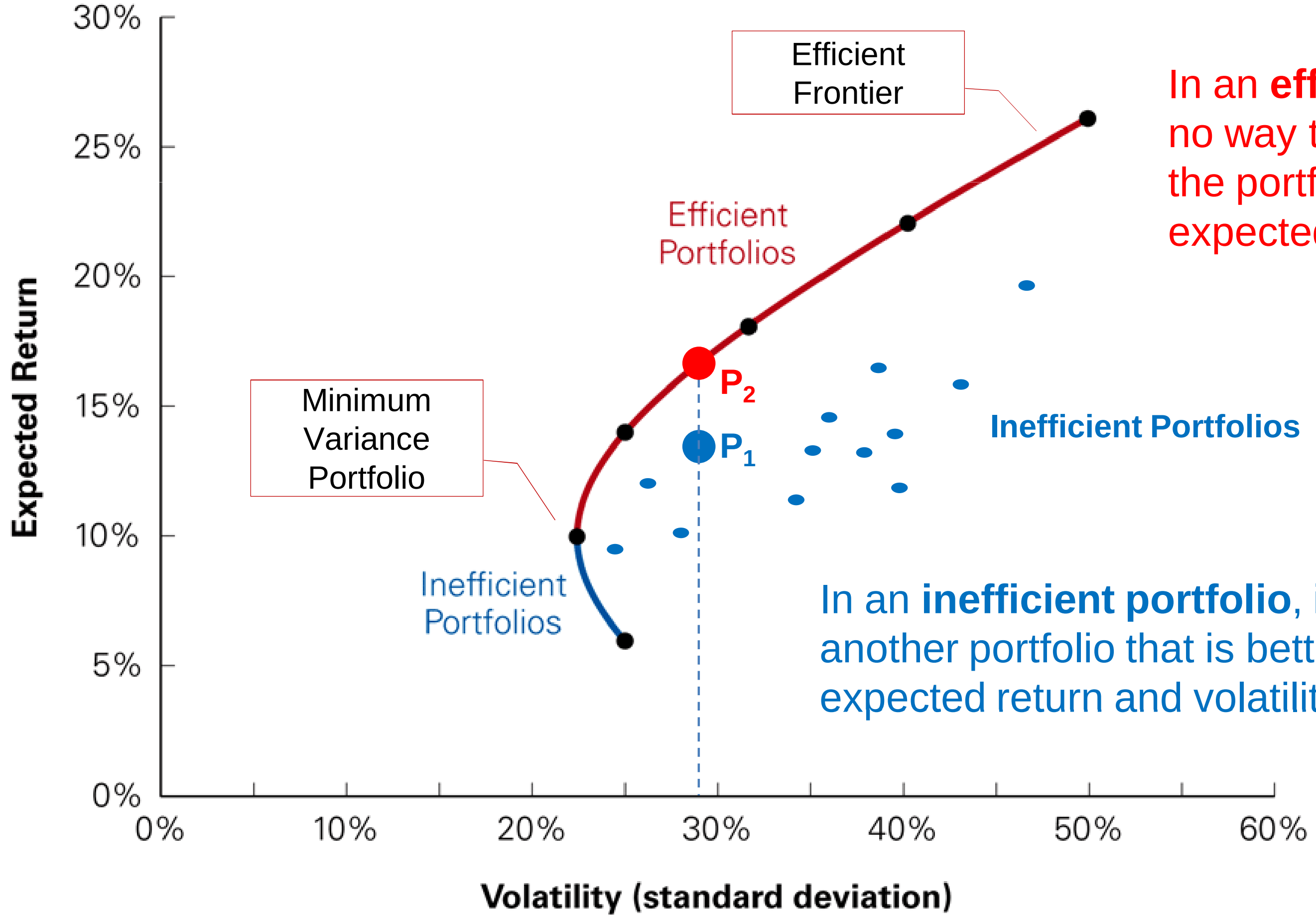
The choice between B and C depends on investor's preferences for return versus risk

The portfolio A is beaten by B (B has a higher expected return and lower volatility)

Example: Your friend has invested 100% of its money in Coca-Cola stock and is seeking investment advice. He would like to earn the highest expected return possible without increasing the risk (volatility). Which portfolio would you recommend?

Efficient Portfolio: Main Characteristics

Volatility Versus Expected Return for Portfolios of Intel and Coca-Cola Stock



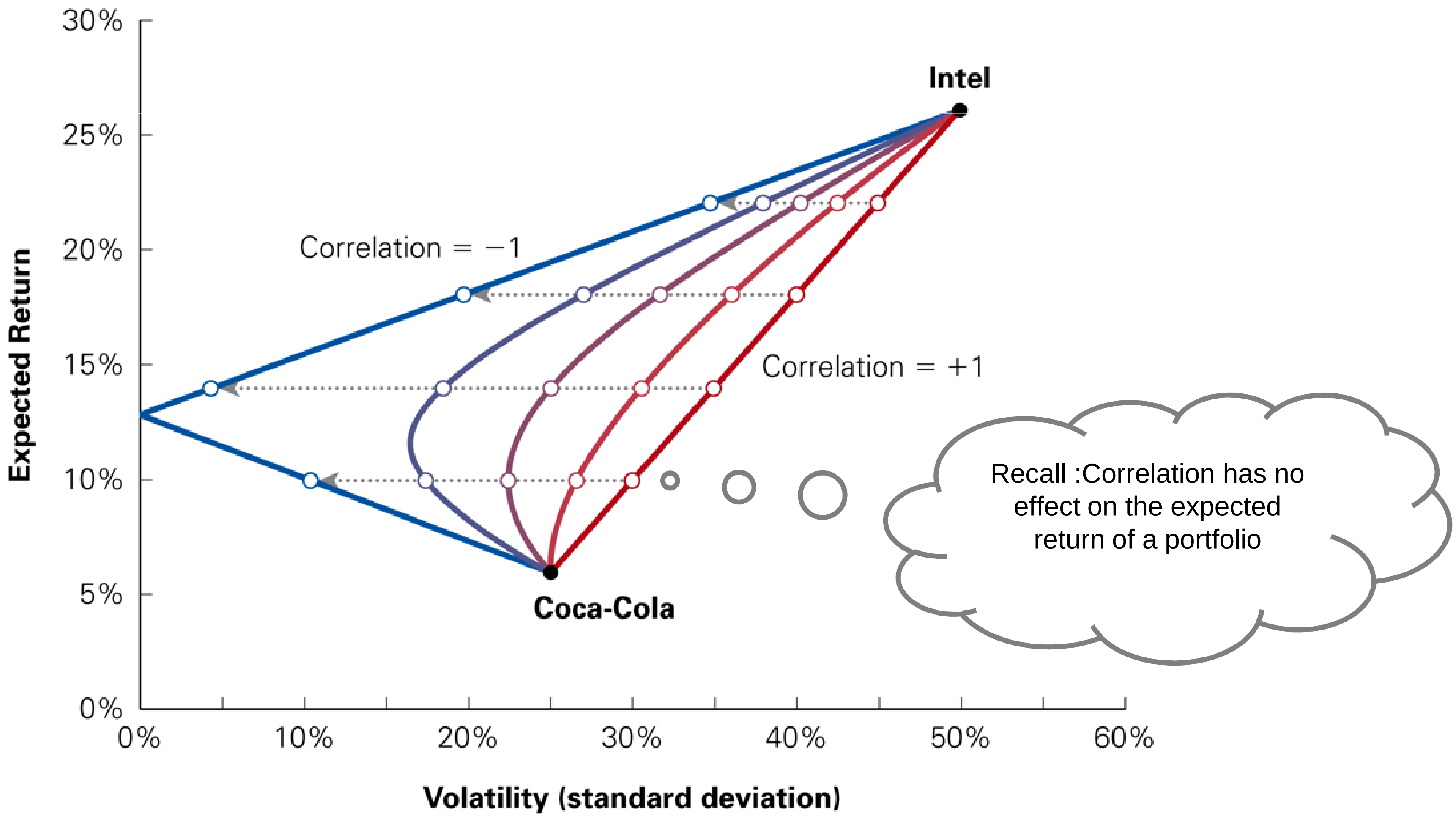
In an **efficient portfolio** there is no way to reduce the volatility of the portfolio without lowering its expected return

In an **inefficient portfolio**, it is possible to find another portfolio that is better in terms of both expected return and volatility.

- Efficient Portfolio: for a given level of risk, it offers the highest possible expected return

The Effect of Correlation

Effect on Volatility and Expected Return of Changing the Correlation between Intel and Coca-Cola Stock



- The lower the correlation, the lower the volatility we can obtain

Short Sales: principle

Short sale transaction

In a short sale, you sell a stock that you do not own, **with the obligation** to buy it back in the future.

We can include a **short position** as part of a portfolio by assigning that stock a negative portfolio weight

Short Position in a portfolio		**Long Position**
A negative portfolio weight of that security	≠	A positive portfolio weight of that security

What is the rationale behind short selling?

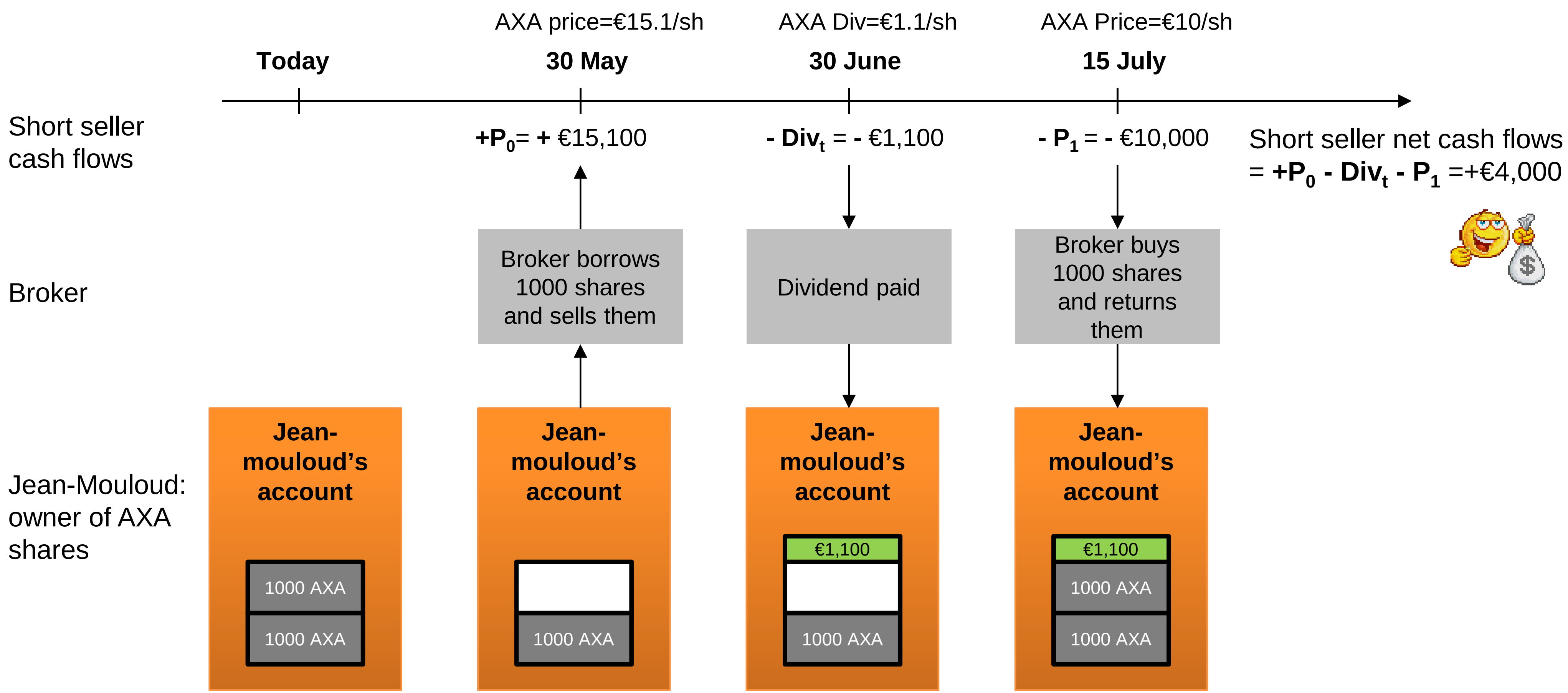
- Short selling is an advantageous strategy if you expect a stock price to decline in the future.

	Date 0	Date t	Date 1
Cash flows from buying a stock	$-P_0$	$+Div_t$	$+P_1$
Cash flows from short selling a stock	$+P_0$	$-Div_t$	$-P_1$

The Mechanics of a Short Sale

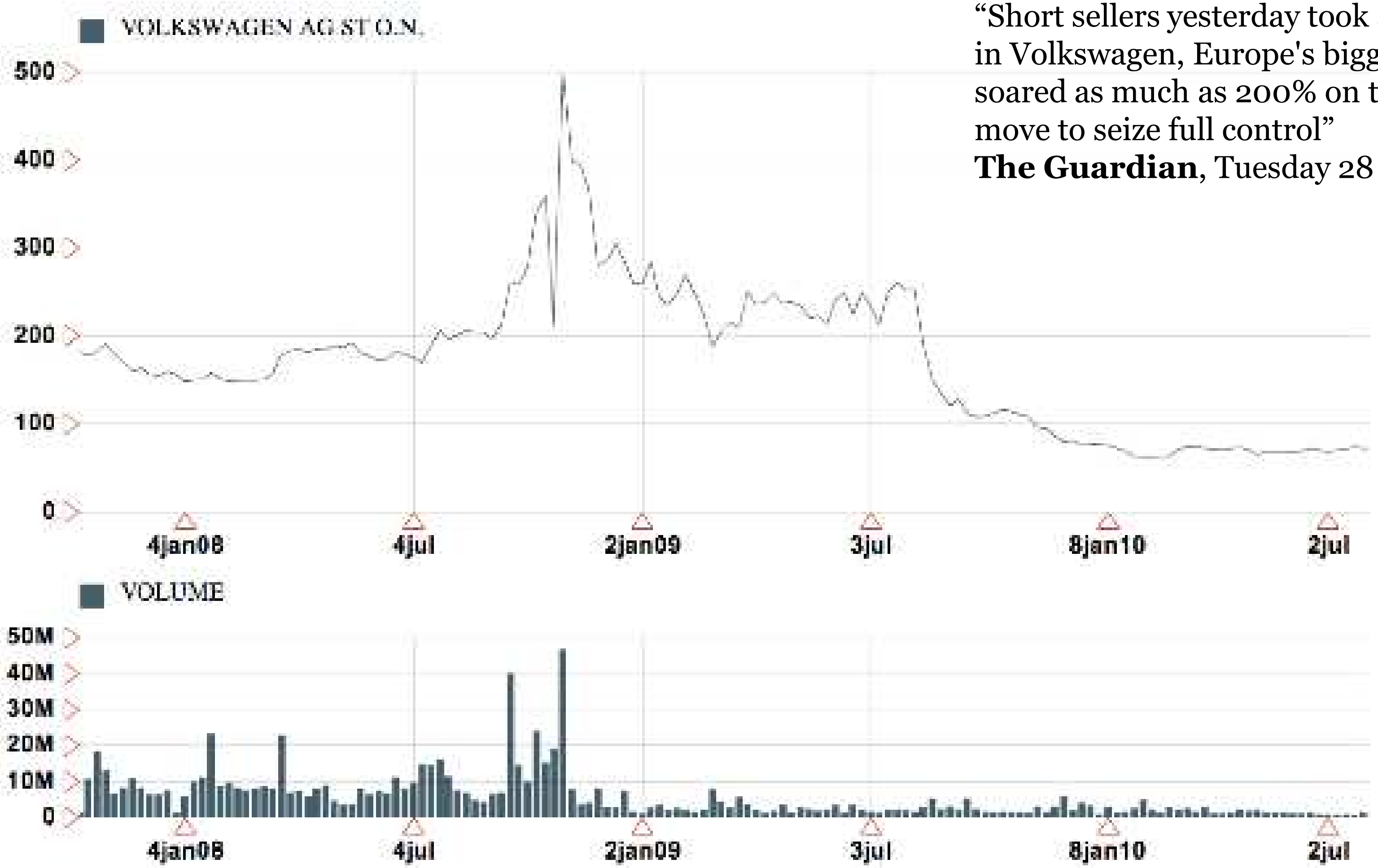
Example: Short sale transaction

Suppose you expect a decrease of the AXA's stock price. You decide to short sell 1000 shares of AXA and to close your position as soon as the AXA's stock price drop to €10.



Short Selling: the case of Volkswagen's Stock in October 2008

VW Shares Plunge, a Day After Surge: the mystery of a 200% increase



"Short sellers yesterday took a pounding as shares in Volkswagen, Europe's biggest carmaker, soared as much as 200% on the back of Porsche's move to seize full control"
The Guardian, Tuesday 28 October 2008

The Mechanics of a Short Sale

Problem: Expected Return and Volatility with a Short Sale

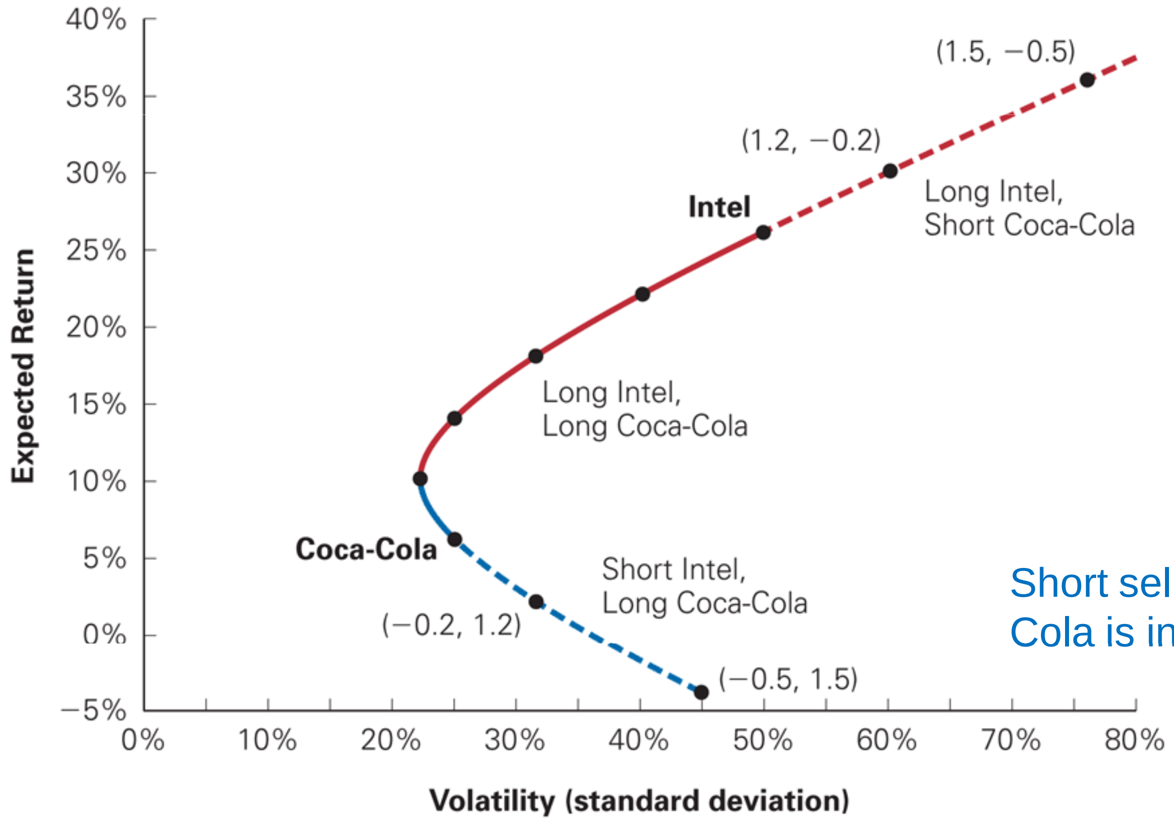
Suppose you have €20,000 in cash to invest. You decide to short sell €10,000 worth of Coca-Cola stock and invest the proceeds from your short sale, plus your €20,000, in Intel. What is the expected return and volatility of your portfolio?

	Expected Return	Volatility	Cov (Intel, Coca)
Intel	26%	50%	0
Coca-Cola	6%	25%	

- In this case, Short selling ………………. the expected return of your portfolio, but also its volatility, above those of the individual stocks

Efficient Portfolios Allowing for Short Sales

Portfolios of Intel and Coca-Cola Allowing for Short Sales



Short selling Coca-Cola to invest in Intel is efficient and might be attractive to an aggressive investor who is seeking high expected return

Short selling Intel to invest in Coca-Cola is inefficient

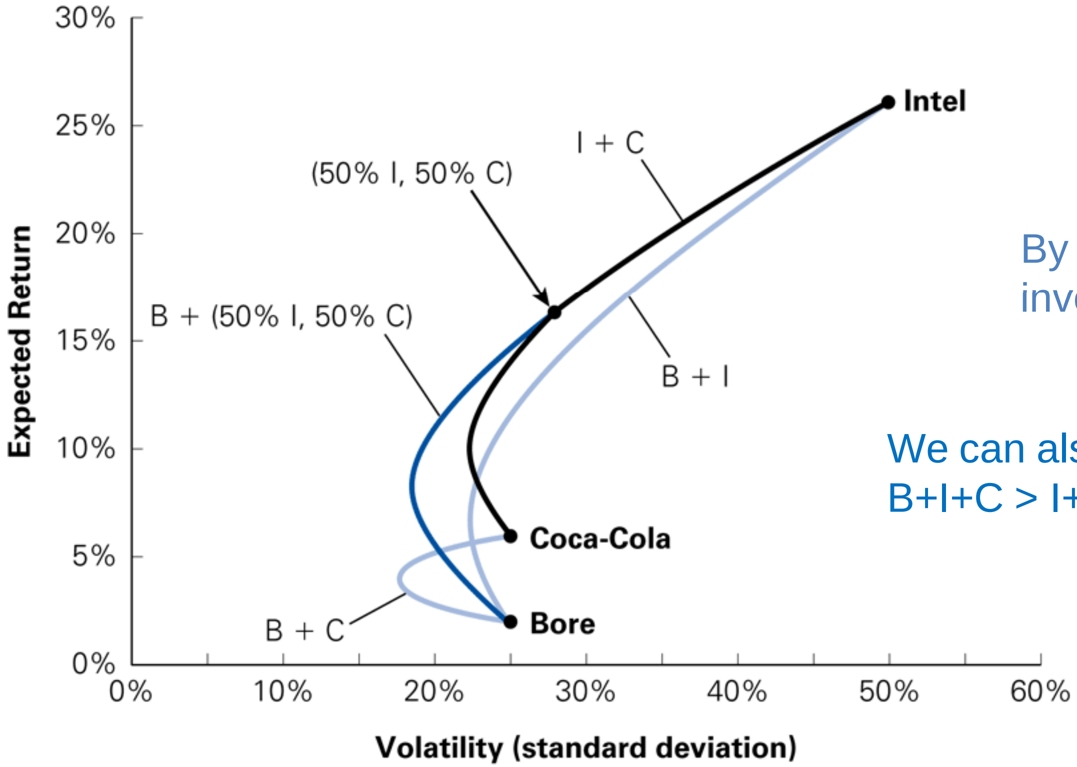
- Short selling extents investment possibilities for investors

Efficient Portfolios with Many Stocks

Consider adding Bore Industries to the two stock portfolio:

Bore is uncorrelated with Intel and Coca-Cola, but is expected to have a very low return of 2%, and the same volatility as Coca-Cola. **Should you add Bore in the portfolio Intel-Coca?**

			Correlation with		
Stock	**Expected Return**	**Volatility**	**Intel**	**Coca-Cola**	**Bore Ind.**
Intel	26%	50%	1.0	0.0	0.0
Coca-Cola	6%	25%	0.0	1.0	0.0
Bore Industries	2%	25%	0.0	0.0	1.0

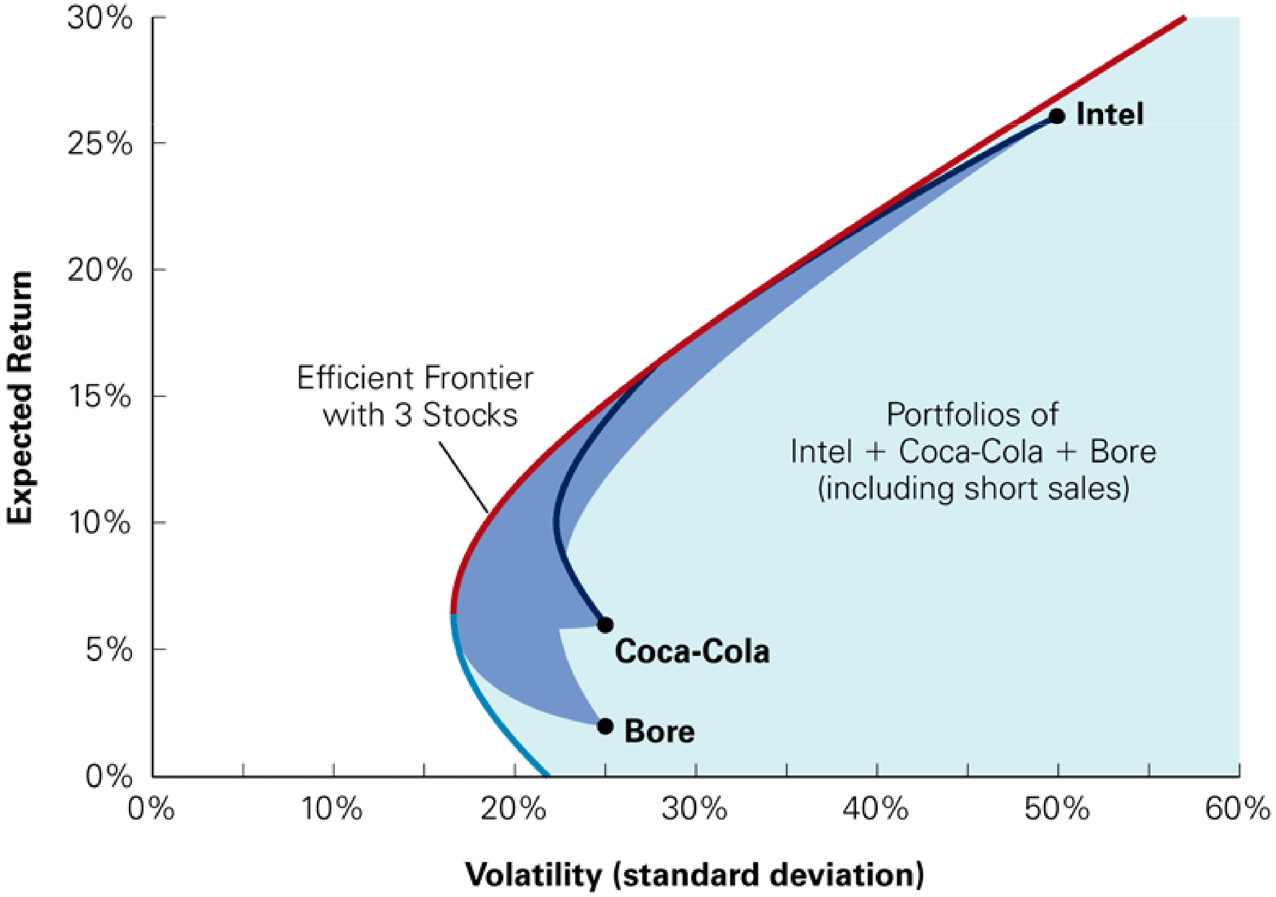


By adding Bore, we introduce new investment possibilities

We can also do better than two-stock portfolios: B+I+C > I+C

Efficient Frontier with Many Stocks

The Volatility and Expected Return for All Portfolios of Intel, Coca-Cola, and Bore Stock

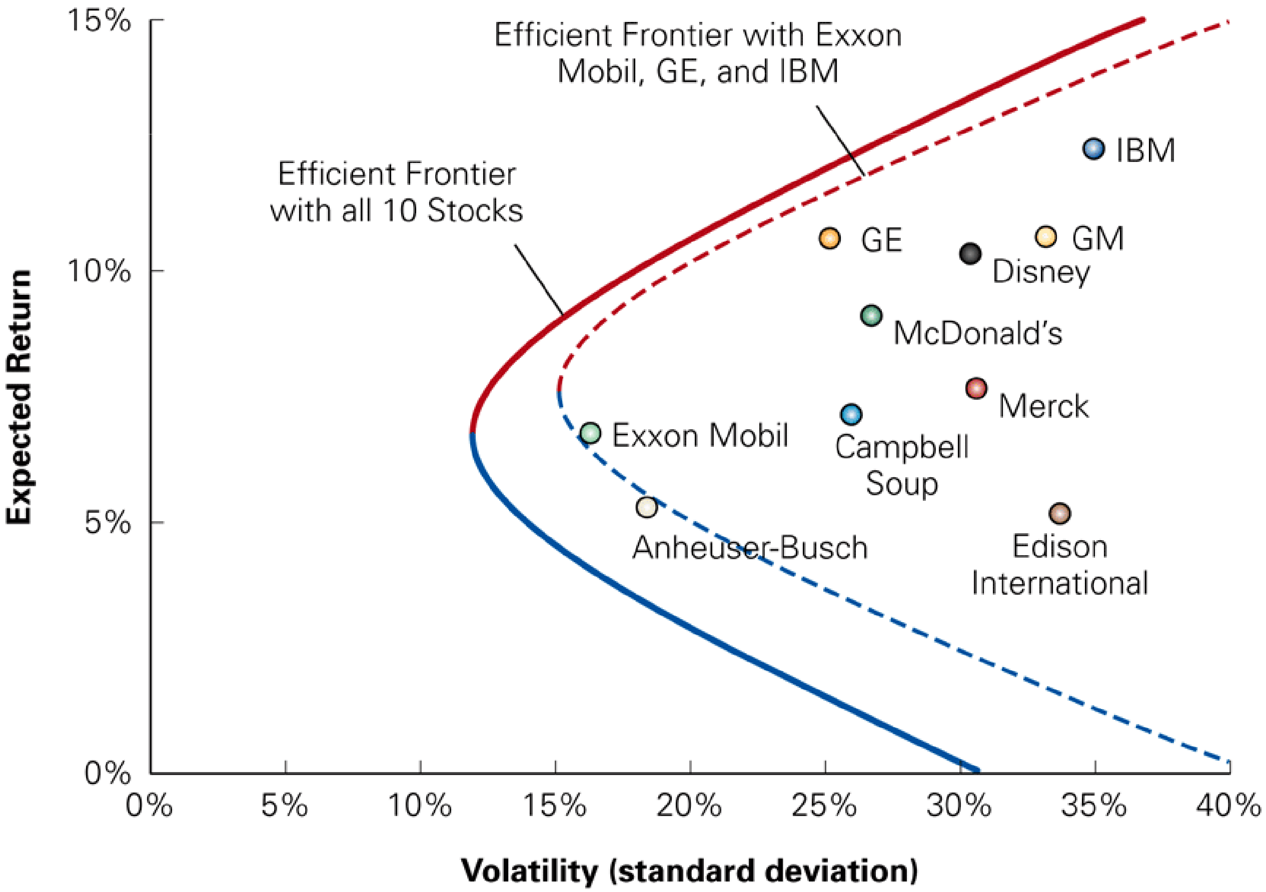


Source : Berk J. and DeMarzo P. (2011), Corporate Finance, Second Edition. Pearson Education. (Figure 11.7 p.348)

- In this case none of the stocks, on its own, is on the efficient frontier, so it would not be efficient to put all our money in a single stock.

Efficient Frontier and the Effect of Diversification

Efficient Frontier with Ten Stocks Versus Three Stocks



Source : Berk J. and DeMarzo P. (2011), Corporate Finance, Second Edition. Pearson Education. (Figure 11.8 p.349)

- In general, adding new investment opportunities allows for greater diversification and improves the efficient frontier

Investing in Risk-Free Securities

Is it profitable to include non-risky investments in our risky-portfolio?

Consider two types of investments:

	Return rate in one year			
Security	Sunny weather	Rainy weather	**Expected Return E[R]**	**Volatility**
Risk-free bond	+3%	+3%	**3%**	**0%**
Umbrella SA	-10%	+20%	**10%**	**20%**

You are planning to invest €100,000 in Umbrella's securities. A friend suggests you to invest a fraction of your money in risk-free bonds. What is the effect on return and risk of investing €50,000 in Umbrella's Securities, while leaving the remaining €50,000 in risk-free bonds?

		Cash flows in one year				
Security	Amount invested	Sunny weather	Rainy weather	Average expected cash flows	**Expected Return E[R]**	**Volatility**
Risk-free bond	€50,000	€51,500	€51,500	€51,500	**3%**	**0%**
Umbrella SA	€50,000	€45,000	€65,000	€55,000	**10%**	**20%**
Total	€100,000	€95,500	€116,500	€106,500	**6.5%**	**10%**

- Risk can be reduced by investing a portion of a portfolio in a risk-free investment, like T-Bills. However, doing so will likely reduce the expected return

Investing in Risk-Free Securities

Let's generalize

Consider an arbitrary risky portfolio (with returns R_p) and the effect on risk and return of putting a **fraction *x* of the money in the portfolio**, while leaving the remaining fraction in risk-free Treasury bills with a yield of r_f. What is the expected return and the volatility of the total portfolio?

Expected Return

$$E[R_P] = x_1.E[R_1] + x_2.E[R_2]$$

$$E[R_{xP}] = (1-x).r_f + x.E[R_P]$$

$$\Rightarrow \boxed{E[R_{xP}] = r_f + x.(E[R_P] - r_f)} \quad (1)$$

Volatility

$$Var(R_P) = x_1^2 Var(R_1) + x_2^2 Var(R_2) + 2x_1 x_2 Cov(R_1, R_2)$$

$$Var(R_{xP}) = (1-x)^2 \underbrace{Var(r_f)}_{0} + x^2 Var(R_P) + 2(1-x)x.\underbrace{Cov(r_f, R_P)}_{0}$$

$$Var(R_{xP}) = x^2 Var(R_P) \Rightarrow \boxed{SD(R_{xP}) = x.SD(R_P)} \quad (2)$$

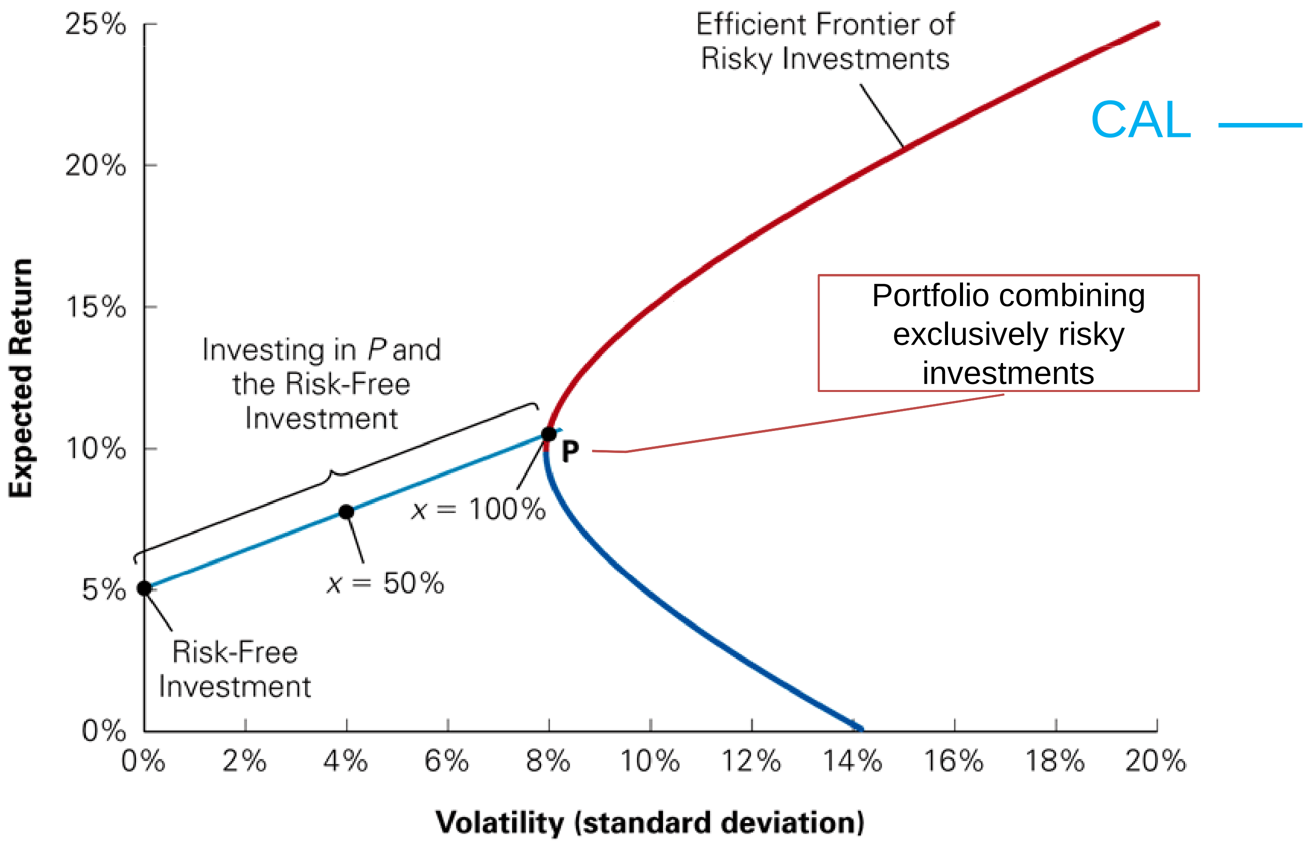
(1) (2)

$$\Rightarrow \boxed{E[R_{xP}] = r_f + \frac{(E[R_P] - r_f)}{SD(R_P)}.SD(R_{xP})}$$

The Capital Allocation Line (CAL)

The Capital Allocation Line

The Risk–Return Combinations from Combining a Risk-Free Investment and a Risky Portfolio



$$E[R_{xP}] = r_f + \frac{(E[R_P] - r_f)}{SD(R_P)} . SD(R_{xP})$$

$$E[R_{xP}] = r_f + x.(E[R_P] - r_f)$$

- As we increase the fraction x invested in P, we increase both our risk and our risk premium proportionally

Buying Stocks Using Leverage: A Levered Portfolio

Is it profitable to borrow money at the risk-free interest rate and invest it in stocks?

Problem: Suppose you have €10,000 in cash, and you decide to borrow another €10,000 at 5% interest rate in order to invest €20,000 in Umbrella's securities (P_U). What is the expected return and risk of your investment? What is your realized return in both sunny or rainy weather?

Security	Return rate in one year		Expected Return E[R]	Volatility
	Sunny weather	Rainy weather		
Umbrella SA	-10%	+30%	**10%**	**20%**

$E[R_{xP}] =$

$SD(R_{xP}) =$

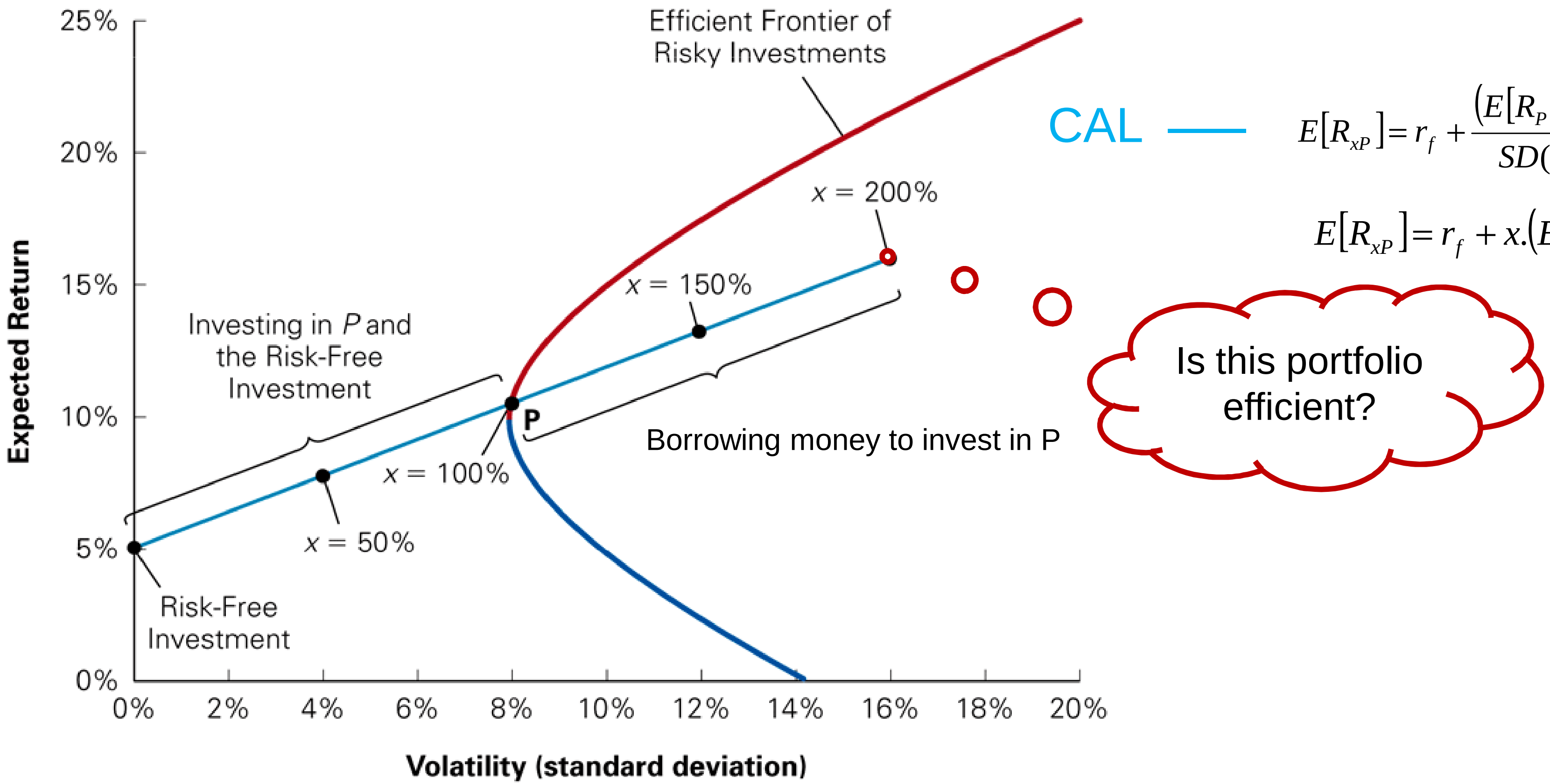
Security	Amount invested	Cash flows in one year		Average expected cash flows	Expected Return E[R]	Volatility
		Sunny weather	Rainy weather			
Loan	-€10,000	-€10,500	-€10,500	-€10,500	**-5%**	**-**
Umbrella SA	€20,000	€18,000	€26,000	€22,000	**10%**	**20%**
Total	€10,000	**€7,500 (-25%)**	**€15,500 (+55%)**	**€11,500**	**15%**	**40%**

- **Using leverage** provided higher expected returns than investing in Umbrella's stocks using only the funds we have available. However, it has doubled the risk of the portfolio

Levered Investment is a Risky Investment Strategy !

The Capital Allocation Line

The Risk–Return Combinations from Combining a Risk-Free Investment and a Risky Portfolio



CAL

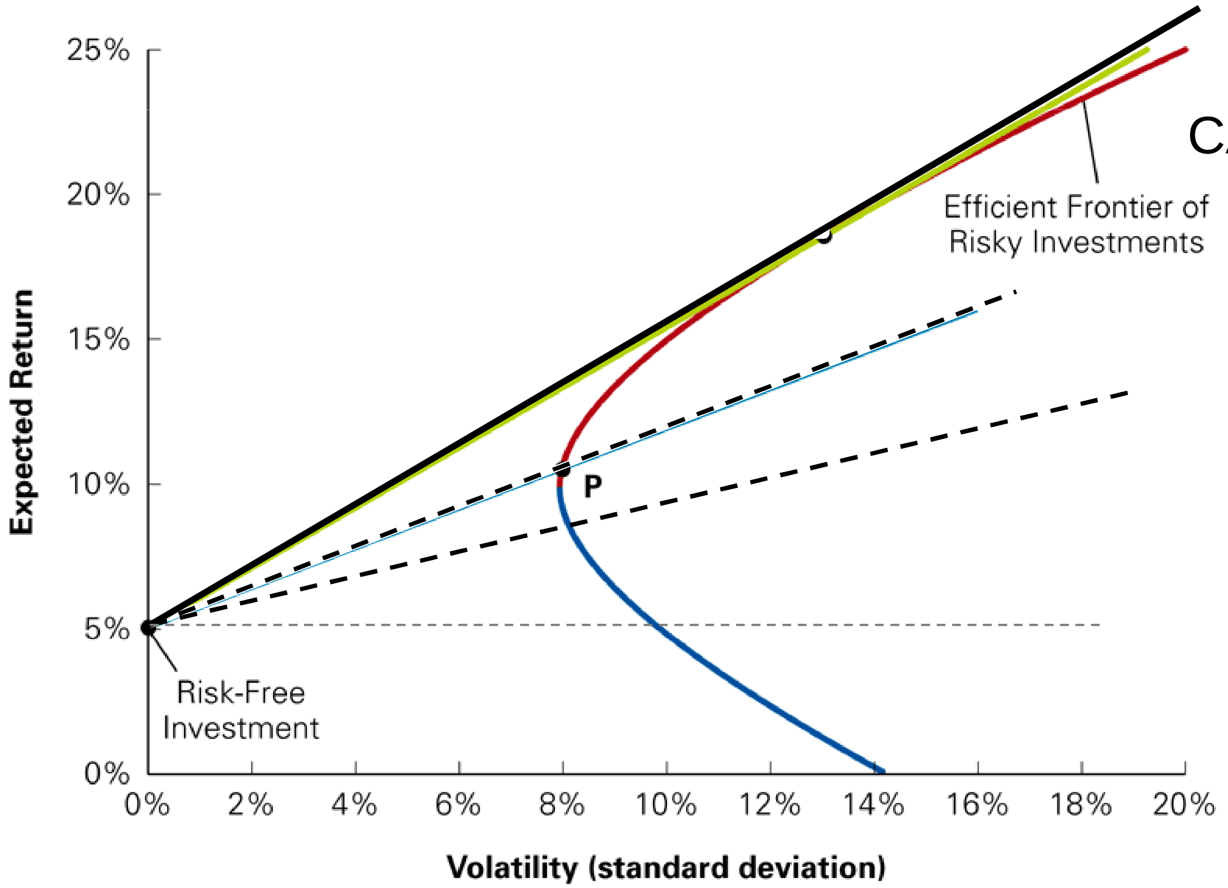
$$E[R_{xP}] = r_f + \frac{(E[R_P] - r_f)}{SD(R_P)} . SD(R_{xP})$$

$$E[R_{xP}] = r_f + x.(E[R_P] - r_f)$$

- The levered portfolio can provide higher expected returns than investing in P using only the funds we have available. However, it has much higher risk …

How to Identify The Optimal Risky Portfolio?

To earn the highest possible expected return for any level of volatility we must find the portfolio that generates the steepest possible line when combined with the risk-free investment.



CAL —— $E[R_{xP}] = r_f + \frac{(E[R_P] - r_f)}{SD(R_P)} . SD(R_{xP})$

The slope of this line is often referred to as the **Sharpe ratio** of the portfolio:

$$\text{Sharpe Ratio} = \frac{\text{Portfolio Excess Return}}{\text{Portfolio Volatility}}$$

$$\text{Sharpe Ratio} = \frac{E[R_P] - r_f}{SD(R_P)}$$

- Sharpe Ratio measures the ratio of **reward-to-volatility provided by a portfolio**

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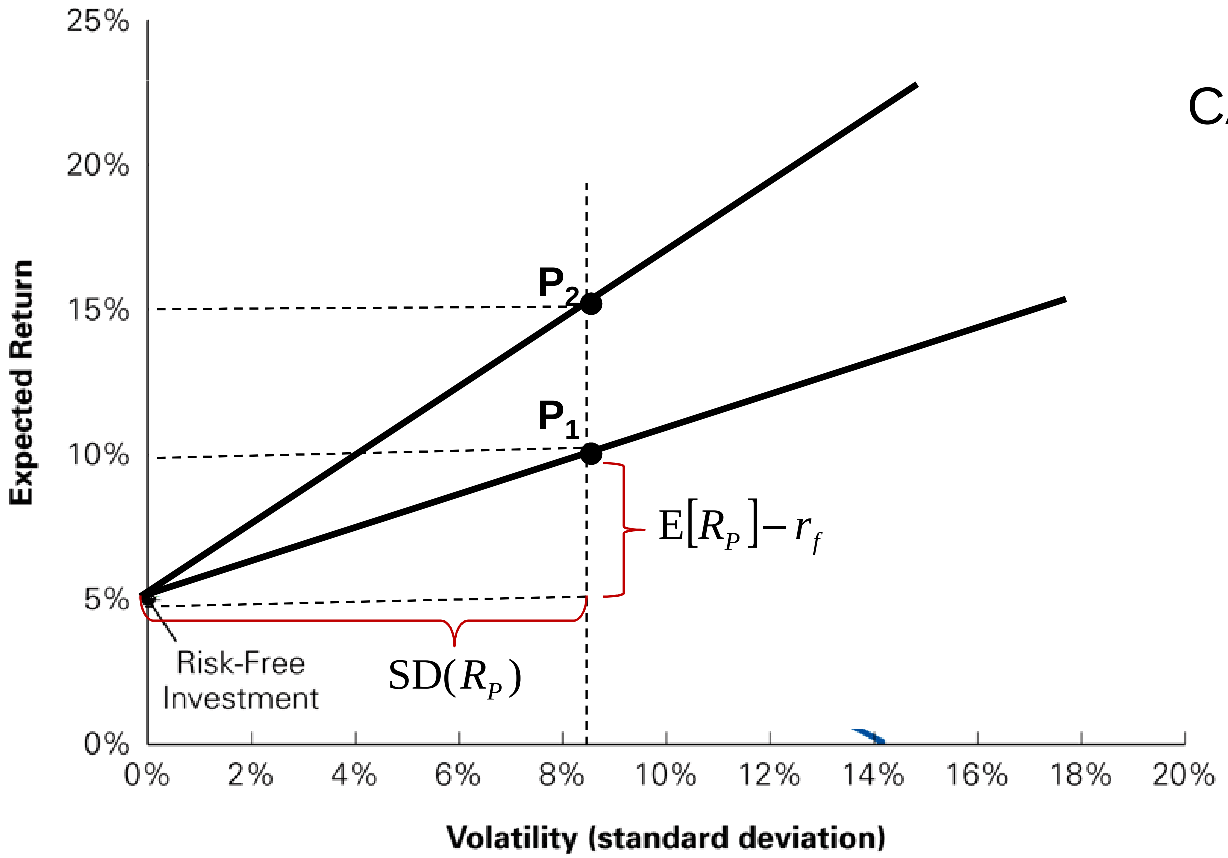

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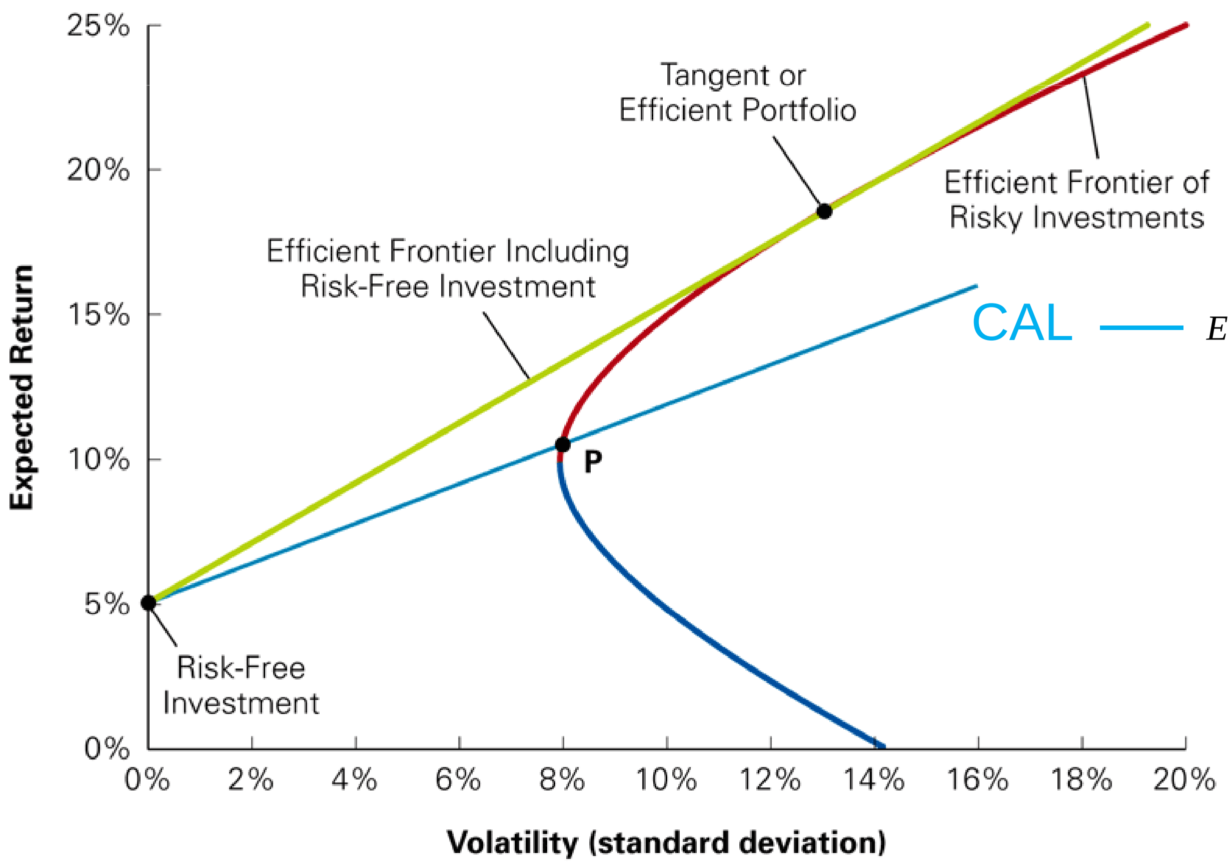


CAL — $E[R_{xT}] = r_f + \frac{(E[R_T] - r_f)}{SD(R_T)}.SD(R_{xT})$

CAL — $E[R_{xP}] = r_f + \frac{(E[R_P] - r_f)}{SD(R_P)}.SD(R_{xP})$

- The portfolio with the highest Sharpe ratio is the portfolio where the line with the risk-free investment is tangent to the efficient frontier of risky investments. The portfolio that generates this tangent line is known as the **Optimal Risky Portfolio** or **Tangent Portfolio**.

The Bottom Line

Combinations of the risk-free asset and the tangent portfolio provide the best risk and return tradeoff available to an investor

- The tangent portfolio is efficient and all efficient portfolios are combinations of the risk-free investment and the tangent portfolio.
- Every investor should invest in the tangent portfolio ***independent of his or her taste for risk***

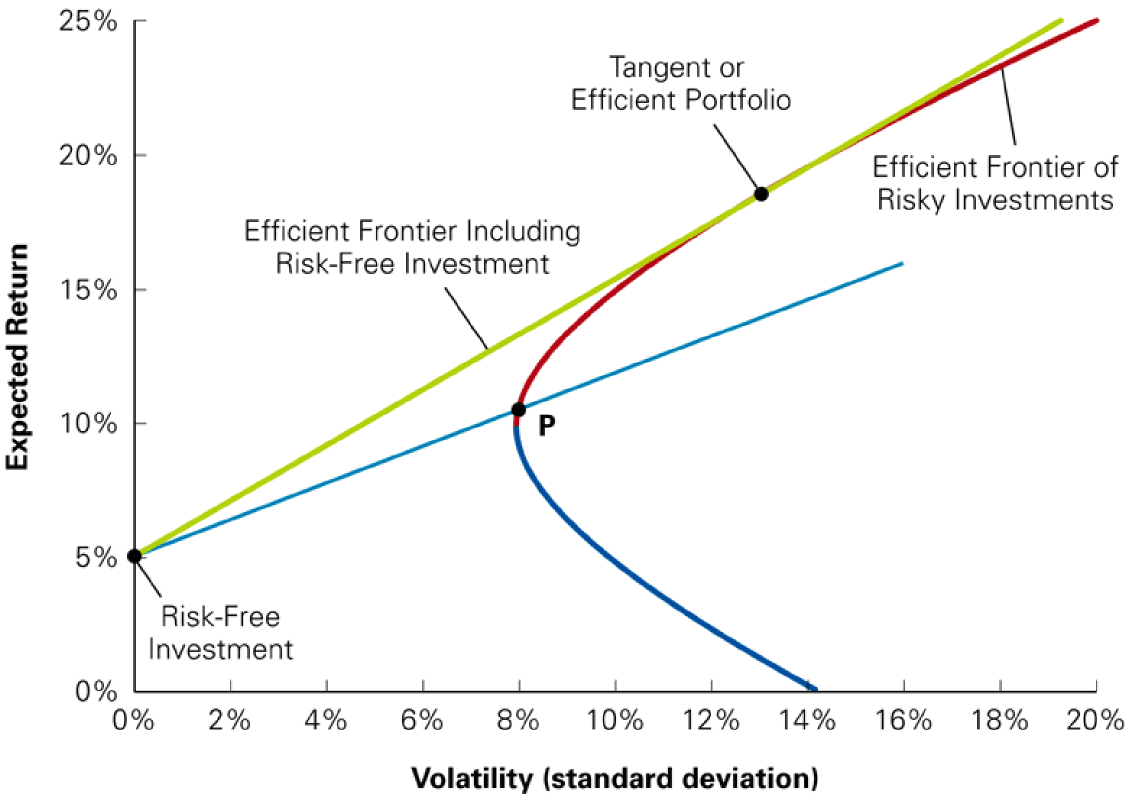
An investor's preferences will determine only *how much* to invest in the tangent portfolio versus the risk-free investment.

- Conservative investors will invest a small amount in the tangent portfolio.
- Aggressive investors will invest more in the tangent portfolio.
- Both types of investors will choose to hold the same portfolio of risky assets, the tangent portfolio, which is the **optimal risky portfolio**

Optimal Portfolio Choice

Problem

Your uncle asks for investment advice. Currently, he has €100,000 invested in portfolio P (see fig. below), which has an expected return of 10.5% and a volatility of 8%. Suppose the risk-free rate is 5%, and the tangent portfolio has an expected return of 18.5% and a volatility of 13%. **1.** To Maximize his expected return without increasing his volatility, which portfolio would you recommend? **2.** If your uncle prefers to keep his expected return the same but minimize his risk, which portfolio would you recommend? **3.** Your uncle asked you also to give him the expected return of an efficient portfolio that has a standard deviation of return of 12%.



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Solution Q.1

Solution Q.2

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Solution Q.3

Portfolio Theory

Harry Markowitz
Nobel Prize in 1990

« Portfolio Selection ». *The Journal of Finance*, Vol. 7, No. 1. (Mar., 1952), pp. 77-91.

- Fundamentals of Portfolio Theory
- Developed techniques of mean-variance portfolio optimization, which allow an investor to find the portfolio with the highest expected return for any level of variance
- Markowitz's Approach has evolved into one of the main methods of portfolio optimization used on Wall Street

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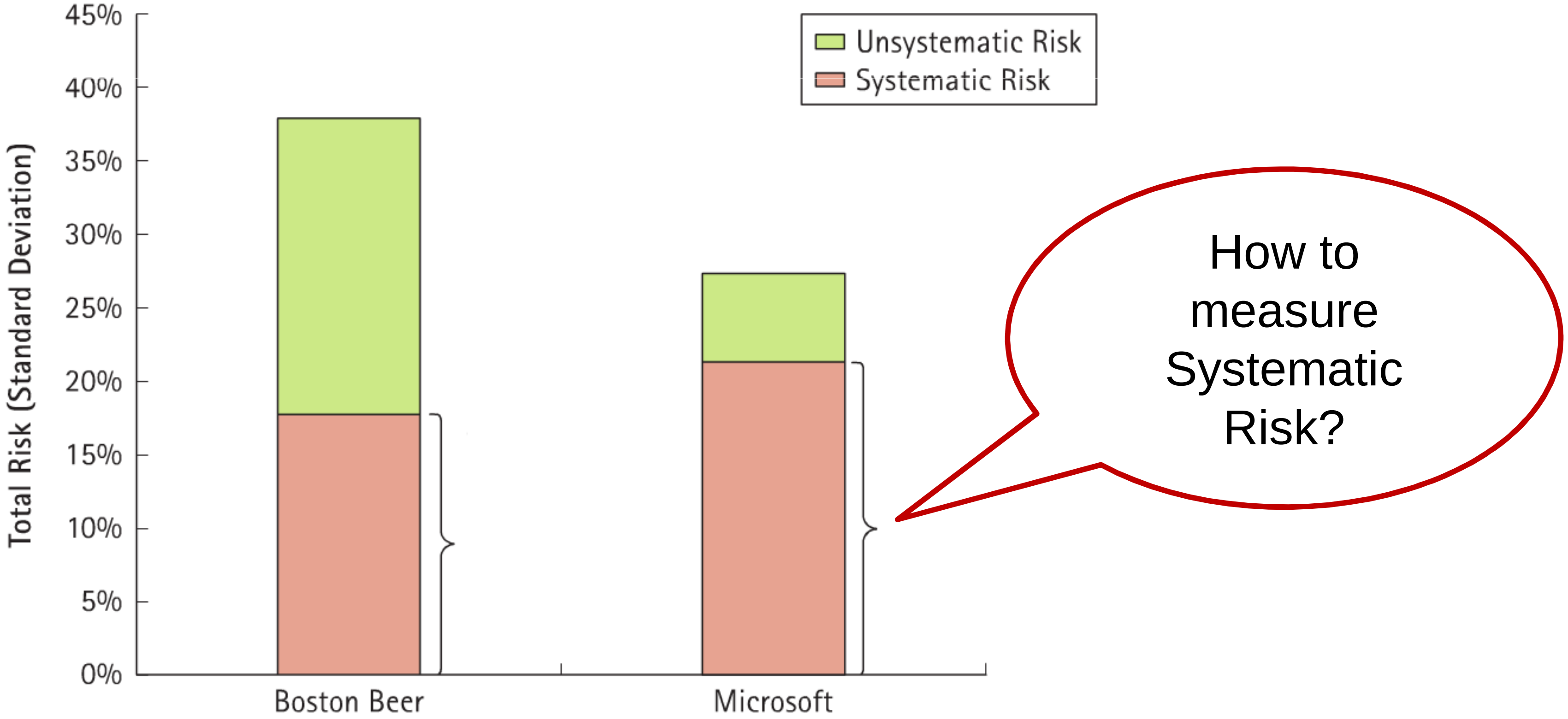
The Capital Asset Pricing Model (CAPM)	The CAPM Assumptions and the Market Portfolio Measuring the Cost of Capital: the CAPM Equation The Capital Market Line Versus The Security Market Line Summary of the CAPM

Recall: Standard Deviation is the Measure of The Total Risk of a Stock

Total Risk of a Stock = Diversifiable Risk + Systematic Risk



The measure of total risk is the Standard Deviation of stock returns

- What is the measure of systematic risk ?

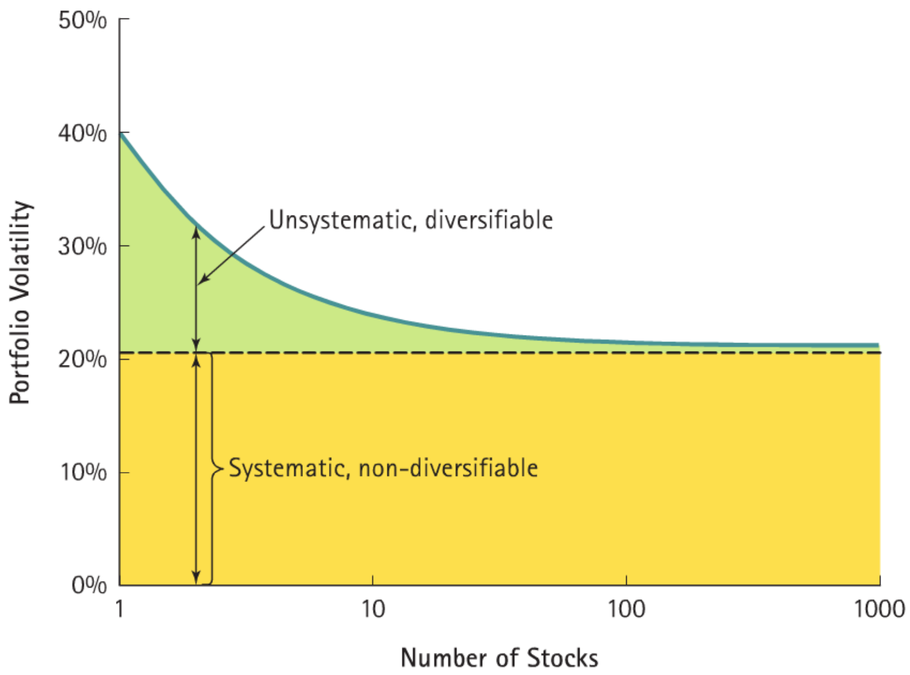
Recall: the Systematic Risk is Non Diversifiable

In a competitive Market…

…investors could eliminate the firm-specific risk “for free” by diversifying their portfolios

- They will not require a reward or risk premium for holding it

No. Stocks	Portfolio Volatility
1	40.0%
2	32.0%
3	28.8%
4	27.1%
5	26.0%
10	23.7%
15	22.9%
20	22.5%
25	22.2%
30	22.1%
50	21.7%
100	21.5%
1000	21.2%



However, diversification does not reduce systematic risk

- **The risk premium of a security is determined by its systematic risk and does not depend on its diversifiable risk**

Measuring Systematic Risk via The Market Portfolio

To measure the systematic risk of a stock …

…we must determine how much of the variability of its return is due to systematic risk

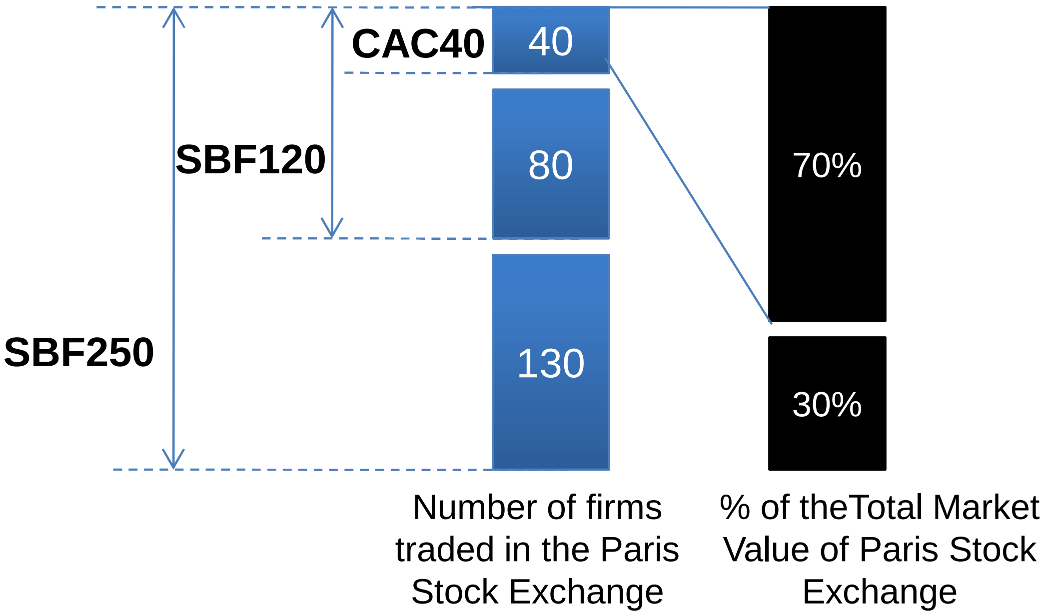
- how sensitive a stock is to systematic shocks that affect the economy as a whole?
- look at the average change in the return for each 1% change in the return of a **portfolio that fluctuates solely due to systematic risk**.

How can we identify such a portfolio – that contains only systematic risk?

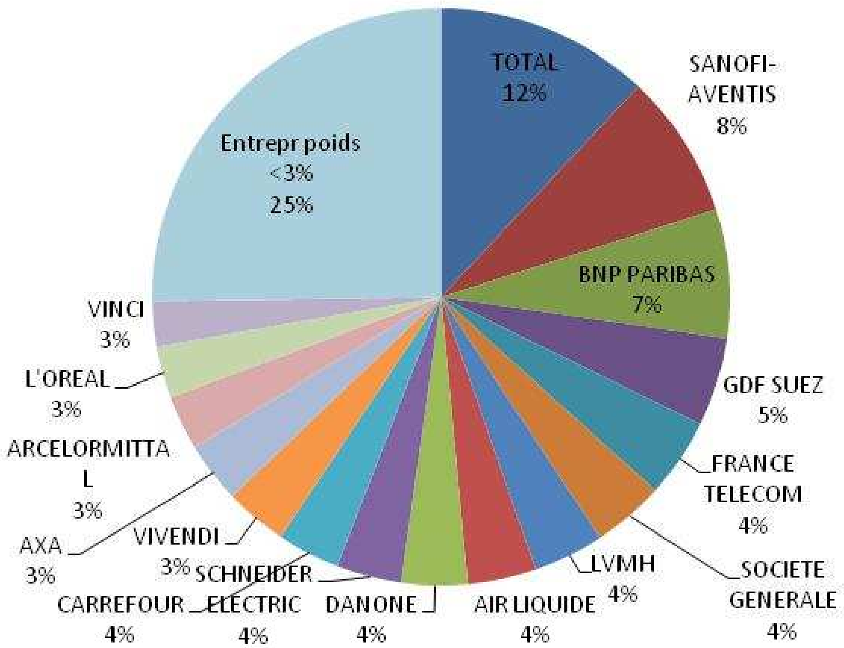
- A fully diversified portfolio
  - A large portfolio containing many different stocks
    - **The Market Portfolio**: which contains the largest number of shares and securities traded in the capital market
    - **In practice,** we use the **CAC40** or the **S&P500 portfolio** as an approximation for the market portfolio

How to Invest in The Market Portfolio?

Paris Stock Market



The weights of main CAC40 securities



A **Market Index** reports the value of a particular portfolio of securities. Example: CAC40, S&P 500, etc.

- The **CAC40** is an index that represents a value-weighted portfolio of 40 of the largest French stocks
- Investing in a Market Index: invest in **index funds** or **ETF** (exchange-traded fund) that invest in market portfolios. Ex. Lyxor ETF CAC40, SPDR (S&P Depository Receipts, nick-named “Spiders”), etc.

How to Invest in The Market Portfolio?

Constructing the market portfolio

Market Capitalization

The total market value of a firm's outstanding shares

$$MV_i = (\text{Number of Shares of } i \text{ Outstanding}) \times (\text{Price of } i \text{ per Share}) = N_i \times P_i$$

Example : Market Capitalisation of EDF (Number of shares = 1.8 billion ; Stock price=17€)

MV(EDF) =

Value-Weighted Portfolio

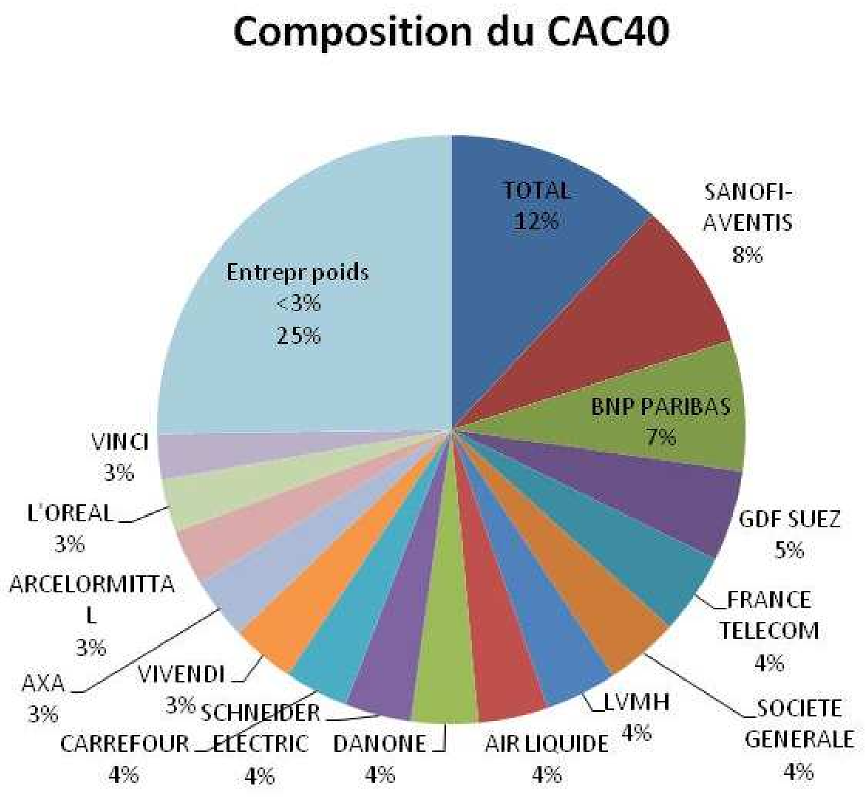
A portfolio in which each security is held in proportion to its market capitalization

$$x_i = \frac{\text{Market Value of } i}{\text{Total Market Value of All Securities}} = \frac{MV_i}{\sum_j MV_j}$$

How to Invest in The Market Portfolio?

Investing €20 000 for constructing a market portfolio

$$x_i = \frac{\text{Market Value of } i}{\text{Total Market Value of All Securities}} = \frac{MV_i}{\sum_j MV_j}$$

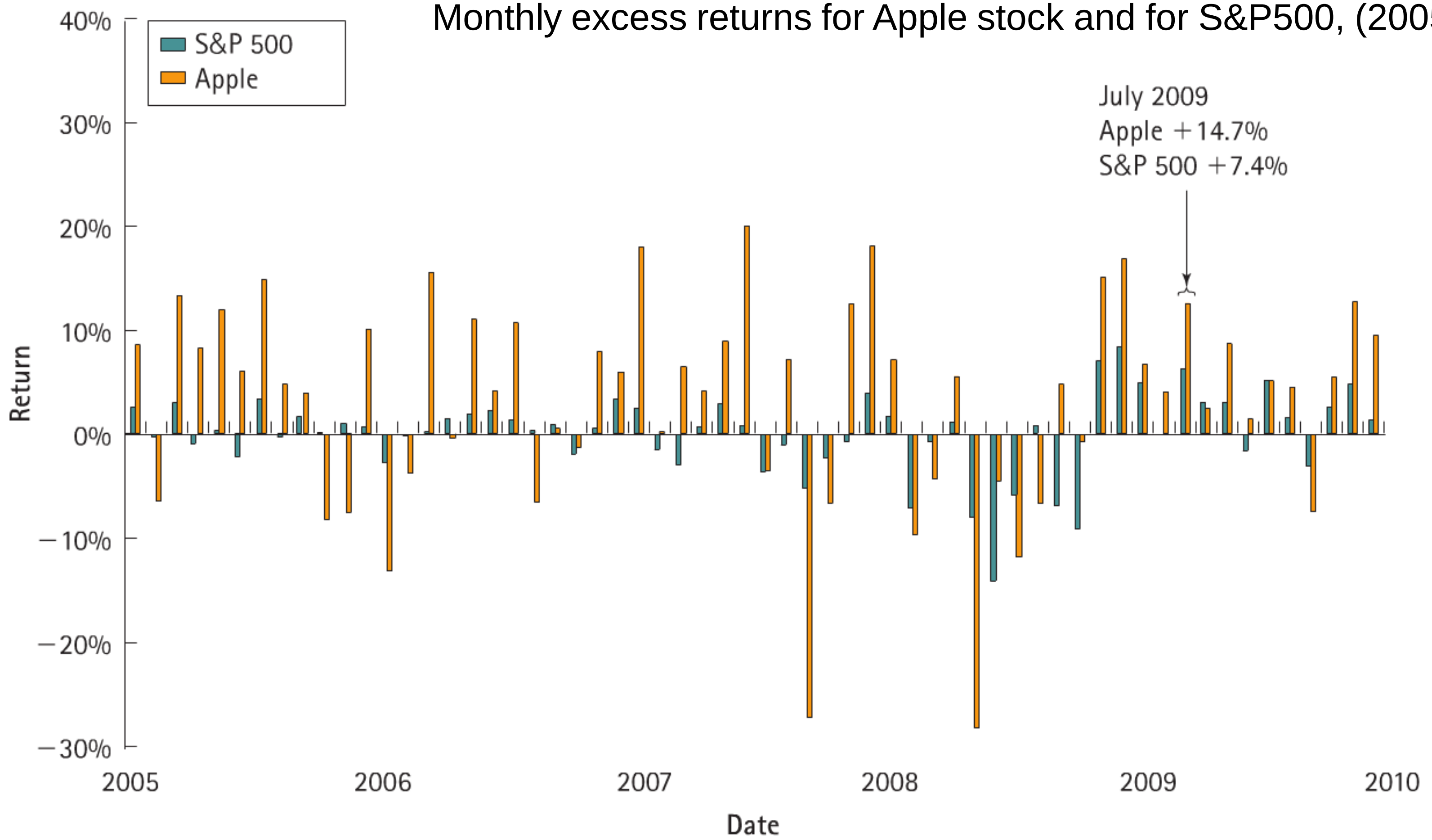


The Market Portfolio is a Good Proxy for Systematic Risk

If we assume that changes in the value of the market portfolio represent systematic shocks to the economy…

- … we can measure the systematic risk of a security by calculating the sensitivity of the security's return to the return of the market portfolio

Monthly excess returns for Apple stock and for S&P500, (2005-2010)



The Beta

If we assume that changes in the value of the market portfolio represent systematic shocks to the economy…

- … we can measure the systematic risk of a security by calculating the sensitivity of the security's return to the return of the market portfolio

Sensitivity to Systematic Risk: Beta (β)

The expected percent change in the excess return of a security for a 1% change in the excess return of the market portfolio.

Quick Chek Question

The Alstom's stock has a β = 2.43. How to interpret this data with respect to CAC40 returns?

- Each 1% change in the return of the market is likely to lead, on average, to a 2.43% change in the return for Alstom
  - The stock of Alstom is exposed to two times more systematic risk than the CAC40

Real-Firm Betas

Betas with respect to the CAC40 for individual stocks (based on monthly data for 2006-2010)

Entreprises	Code (*ticker*)	Bêta	Entreprises	Code (*ticker*)	Bêta
Accor	AC.PA	1,13	L'Oréal	OR.PA	0,95
Air France-KLM	AF.PA	1,99	LVMH	MC.PA	1,12
Air Liquide	AI.PA	0,55	Michelin	ML.PA	0,21
Alcatel-Lucent	ALU.PA	2,76	Pernod Ricard	RI.PA	0,98
Alstom	ALO.PA	2,43	Peugeot	UG.PA	1,69
Arcelor-Mittal	MTP.PA	1,95	PPR	PP.PA	1,13
AXA	CS.PA	– 0,03	Renault	RNO.PA	1,31
BNP Paribas	BNP.PA	1,14	Saint-Gobain	SGO.PA	0,96
Bouygues	EN.PA	0,91	Sanofi-Aventis	SAN.PA	0,49
Capgemini	CAP.PA	2,16	Schneider Electric	SU.PA	0,66
Carrefour	CA.PA	0,69	Société Générale	GLE.PA	1,25
Crédit Agricole	ACA.PA	0,53	STMicroelectronics	STM.PA	1,78
Danone	BN.PA	0,59	Suez	SZE.PA	1,47
Dexia	DX.PA	1,26	Total	FP.PA	0,54
EADS	EAD.PA	1,36	Unibail-Rodamco	UL.PA	0,37
Essilor	EF.PA	0,22	Vallourec	VK.PA	1,10
France Télécom	FTE.PA	1,61	Véolia Environnement	VIE.PA	1,23
Lafarge	LG.PA	0,85	Vinci	DG.PA	0,38
Lagardère	MMB.PA	1,04	Vivendi	VIV.PA	1,40

- Each 1% change in the return of the CAC40 is likely to lead, on average, to a 1.99% change in the return for Air France, but only a 0.54% change in the return for Total

The Beta Versus the Standard Deviation

Standard Deviation: SD → **Systematic Risk + Diversifiable Risk**

The beta : β → **Systematic Risk**

Example

Michelin and Air France have similar standard deviation for 2002-2007 (SD ~ 45%). However, the beta of Air France is higher than the beta of Michelin. How to interpret this data ?

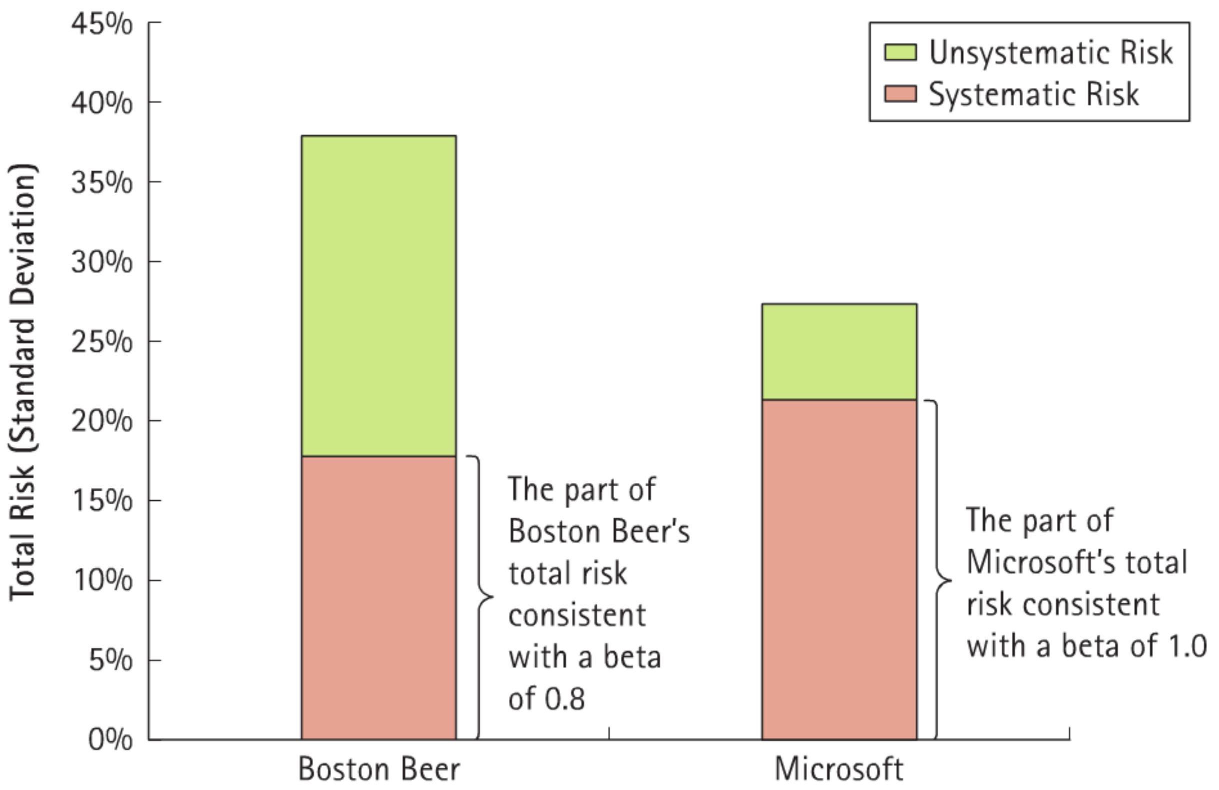
Security	SD	β
Air France-KLM	45%	1.99
Michelin	45%	0.21

➡ Michelin and Air France have similar volatility, but the exposure of Air France to systematic risk is …………….. than Michelin

The Beta Versus the Standard Deviation

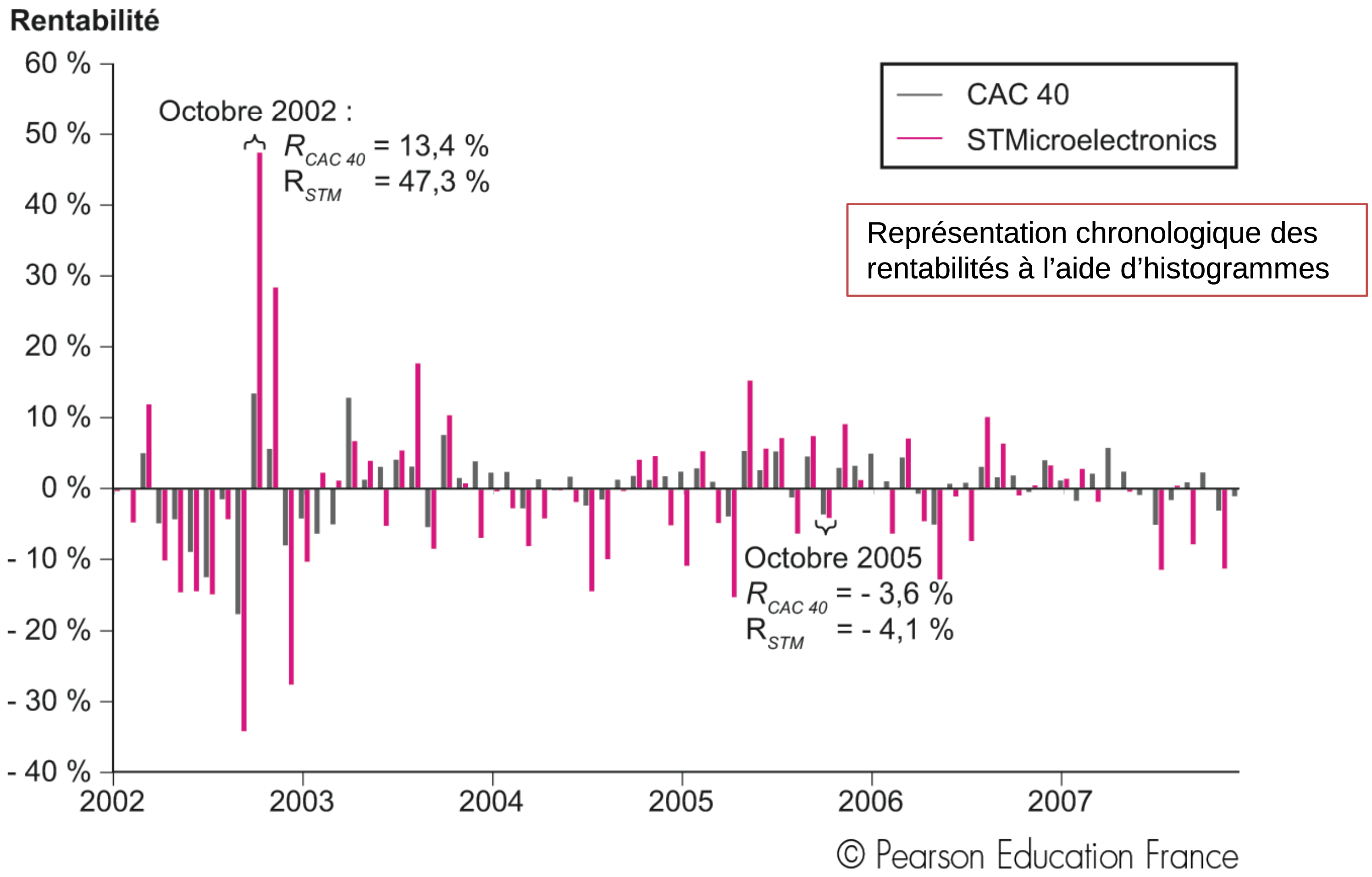
Standard Deviation: SD → **Systematic Risk + Diversifiable Risk**

The beta : β → **Systematic Risk**



Estimating Beta from Historical Returns

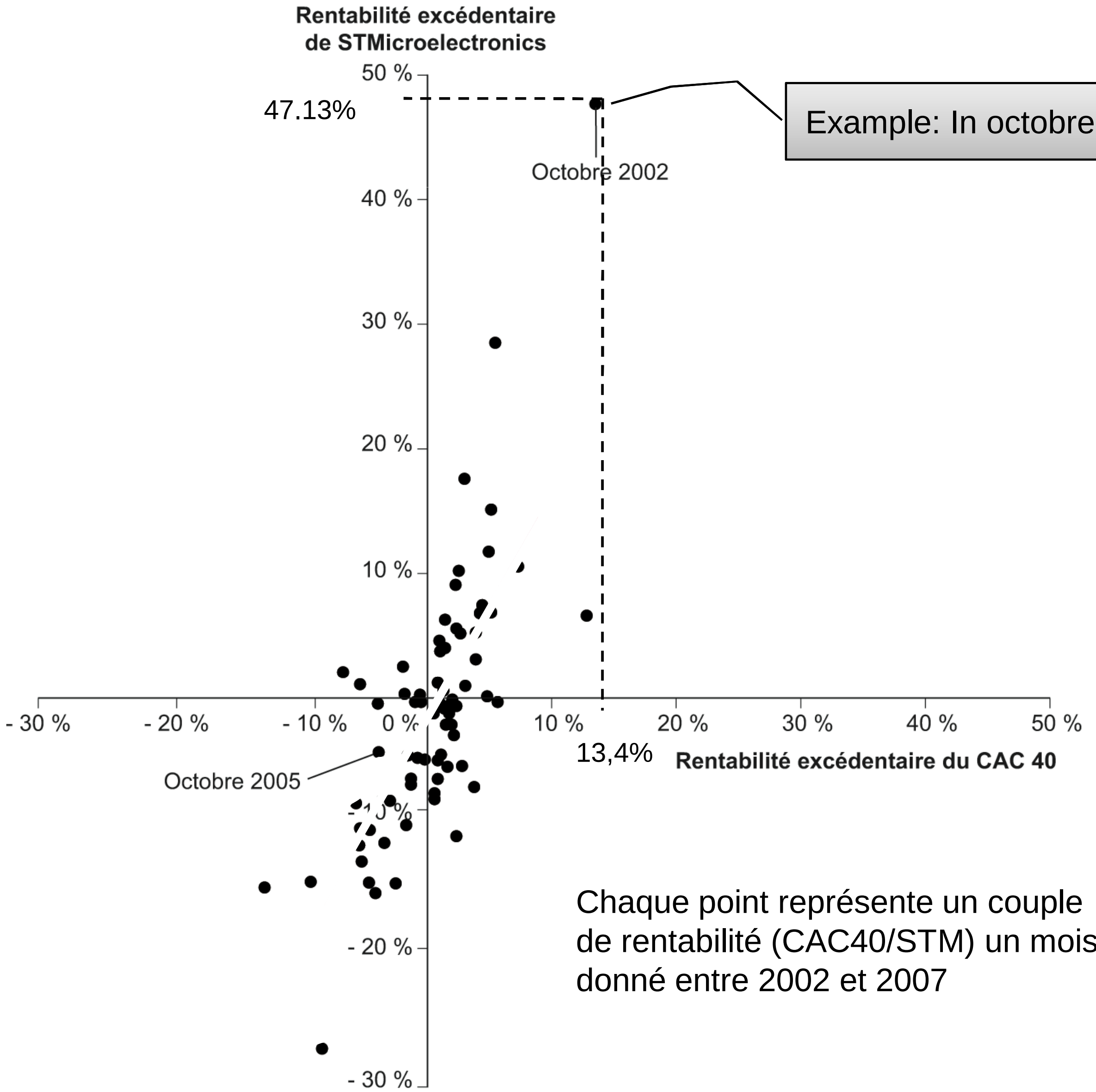
Monthly excess returns for STMicroelectronics and for CAC40, (2002-2007)



➡ STM's returns tend to move in the same direction but farther than those of the CAC40

Estimating Beta from Historical Returns

Monthly excess returns for STMicroelectronics and for CAC40, (2002-2007)

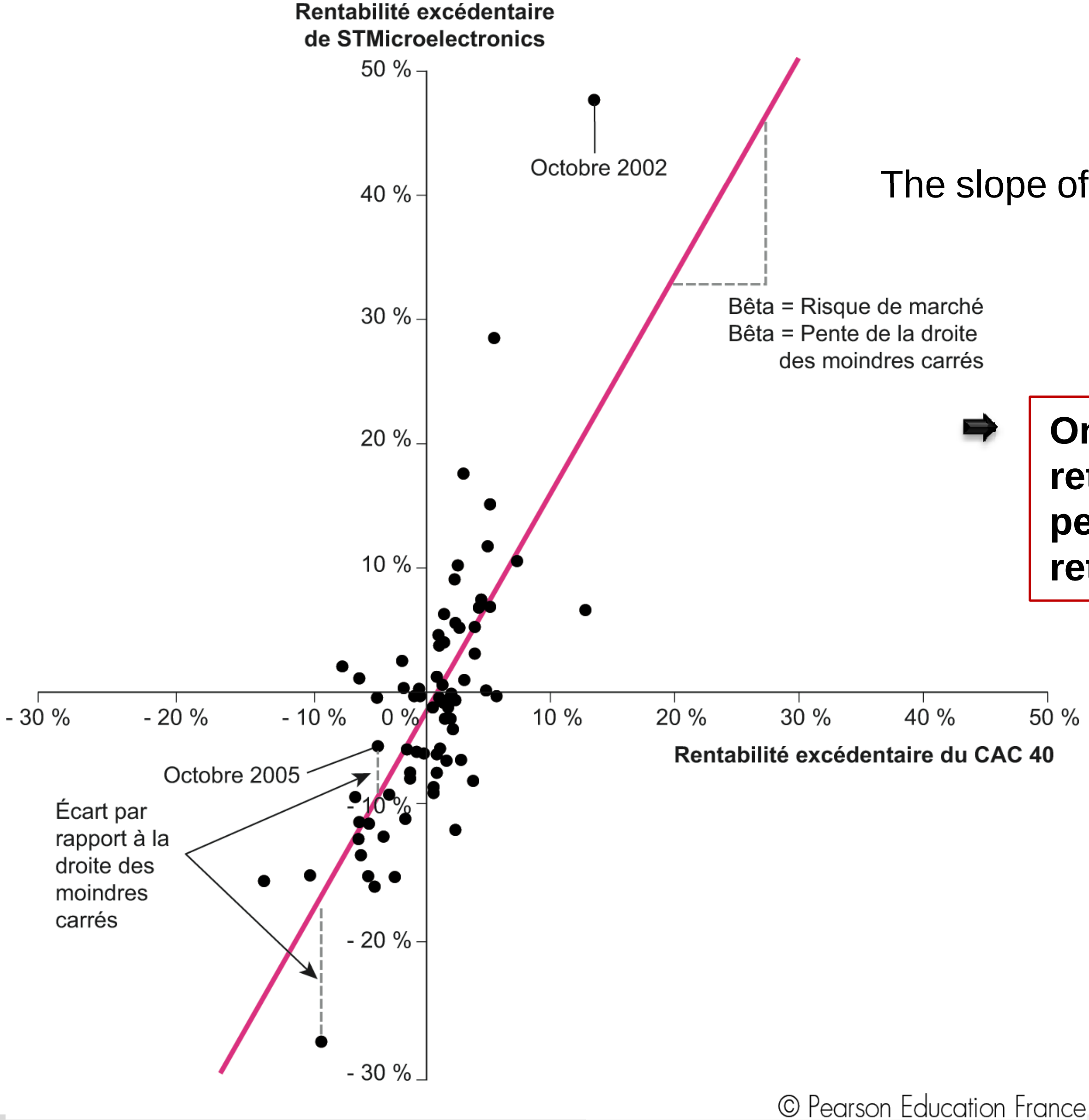


Example: In octobre 2002, R_{STM}=47.13% and R_{CAC40}=13.4%

Chaque point représente un couple de rentabilité (CAC40/STM) un mois donné entre 2002 et 2007

Estimating Beta from Historical Returns

Monthly excess returns for STMicroelectronics and for CAC40, (2002-2007)



The slope of the linear regression line = **1.8 = Beta**

➡ **On average, a 1% change in the return of the CAC40 during this period led to a 1.8% change in the return for STM**

Estimating Beta from Historical Returns

Average Betas for some european stocks based on historical data

$\beta < 0{,}75$		$0{,}75 < \beta < 0{,}85$		$0{,}85 < \beta < 1$		$1 < \beta < 1{,}1$		$\beta \geq 1{,}1$	
Haldengut	-0,13	Benetton	0,76	Rhone Poulenc	1,00	Henkel	1,01	Paribas	1,10
Immobilière Hôtelière	0,24	EMI	0,77	UBS	1,00	Peugeot	1,02	Michelin	1,10
Fromageries Bel	0,36	Air Liquide	0,77	Hoechst	0,99	Lufthansa	1,02	Novartis	1,10
Seita	0,40	Total	0,77	Adecco	0,99	Bayer	1,02	Bank of Scotland	1,11
Eramet	0,45	Club Méditerranée	0,78	Ahold	0,98	BASF	1,02	Credito Italiano	1,11
Hermes	0,49	Elsevier	0,78	Petrofina	0,97	Siemens	1,02	Allianz	1,11
Eurodisney	0,49	British Telecom	0,78	Unilever	0,96	Tabacalera	1,02	Banco Santander	1,11
TF 1	0,54	Reuters	0,78	Preussag	0,96	Vivendi	1,03	Lagardère Groupe	1,15
Skis Rossignol	0,54	Swiss Life	0,78	AXA	0,96	Vodafone	1,03	Volkswagen	1,17
AGF	0,60	Zodiac	0,79	Saint-Gobain	0,96	Deutsche Bank	1,04	BMW	1,17
Eurotunnel	0,64	GIB	0,80	Danone	0,96	Accor	1,04	Telefonica	1,18
Rémy Cointreau	0,65	Generali	0,80	Fiat	0,94	Pinault Printemps	1,05	Legrand	1,19
Canal +	0,66	Clarins	0,81	Nestlé	0,94	Parmalat	1,05	Telecom Italia	1,23
Heineken	0,66	Aguas Barcelona	0,81	Reckitt & Colman	0,93	Italgas	1,06	Pirelli	1,23
Karstadt	0,68	Rolls-Royce	0,82	Abbey National	0,92	Barclays	1,07	Daimler-Benz	1,28
Tesco	0,73	Bouygues	0,83	British Airways	0,89	Société Générale	1,07	Renault	1,30
Marks & Spencer	0,73	Pernod-Ricard	0,83	Veba	0,89	Lafarge	1,07	Dragados	1,31
British Petroleum	0,73	Glaxo Wellcome	0,85	LVMH	0,88	BNP	1,07	Mannesmann	1,33
PolyGram	0,74	SAP	0,85	Elf Aquitaine	0,87	L'Oréal	1,08	Philips	1,33
Castorama	0,74	Shell	0,85	Carrefour	0,87	Cable & Wireless	1,08	Alcatel Alsthom	1,39
Bull	0,75	KLM	0,85	Cadbury Schweppes	0,87	Olivetti	1,09	Schneider	1,44

source : Datastream

Estimating Beta from Historical Returns

Average Betas for stocks by industry and the betas of slected company in each industry (based on historical data)

Industry	Average Beta	Ticker	Company	Beta
Electric Utilities	0.2	EIX	Edison International	0.8
Personal & Household Prods.	0.5	PG	The Procter & Gamble Company	0.6
Food Processing	0.5	HNZ	H. J. Heinz Company	0.6
Restaurants	0.5	SBUX	Starbucks Corporation	1.3
Beverages (Nonalcoholic)	0.6	KO	The Coca-Cola Company	0.6
Retail (Grocery)	0.6	SWY	Safeway Inc.	0.7
Major Drugs	0.7	PFE	Pfizer Inc.	0.7
Beverages (Alcoholic)	0.7	SAM	Boston Beer Company Inc.	0.8
Apparel/Accessories	0.7	ANF	Abercrombie & Fitch	1.6
Retail (Home Improvement)	0.8	HD	Home Depot Inc.	0.7
Software & Programming	0.8	MSFT	Microsoft Corporation	1.0
Recreational Products	1.0	HOG	Harley-Davidson Inc.	2.2
Auto & Truck Manufacturers	1.0	F	Ford Motor Company	2.5
Communications Equipment	1.0	MOT	Motorola	1.7
Forestry & Wood Products	1.0	WY	Weyerhaeuser Company	1.5
Computer Services	1.1	GOOG	Google	1.1
Computer Hardware	1.2	AAPL	Apple	1.5
Conglomerates	1.4	GE	General Electric Company	1.6
Semiconductors	1.5	INTC	Intel Corporation	1.1

Source: Reuters, June 2010.

Estimating Beta from Covariance

If we assume that the Market Portfolio is represented by the CAC40 index, and the beta of a security i is **β_i =1.5**

- Each 1% change in the return of the CAC40 is likely to lead, on average, to a 1.5% change in the return for the security i

- $$\beta_i = \frac{1.5\%}{1\%} = \frac{\text{Variation of } R_i \text{ ralative to variation of } R_{Mkt}}{\text{Variation of } R_{Mkt}} = \frac{Cov(R_i, R_{Mkt})}{Var(R_{Mkt})}$$

- $$\beta_i = \frac{Cov(R_i, R_{Mkt})}{[SD(R_{Mkt})]^2}$$

$$Corr(R_i, R_j) = \frac{Cov(R_i, R_j)}{\sigma_{R_i} \cdot \sigma_{R_j}}$$

Volatility of i that is common with the market

$$\beta_i = \frac{SD(R_i) \cdot Corr(R_i, R_{Mkt})}{SD(R_{Mkt})}$$

Interpreting Beta

A security's beta is related to how sensitive its underlying revenues and cash flows are to general economic conditions

If $\beta_i = 1$ ➡ The security *i* tend to move in the same direction than the market

If $\beta_i > 1$ ➡ The security i is likely to be more sensitive to systematic risk than the market

Shocks in the economy have an amplified impact (negatively or positively) on this stock

If $\beta_i < 1$ ➡ The security i is likely to be more stable and less sensitive to systematic risk than the market

Shocks in the economy on this stock are dampened

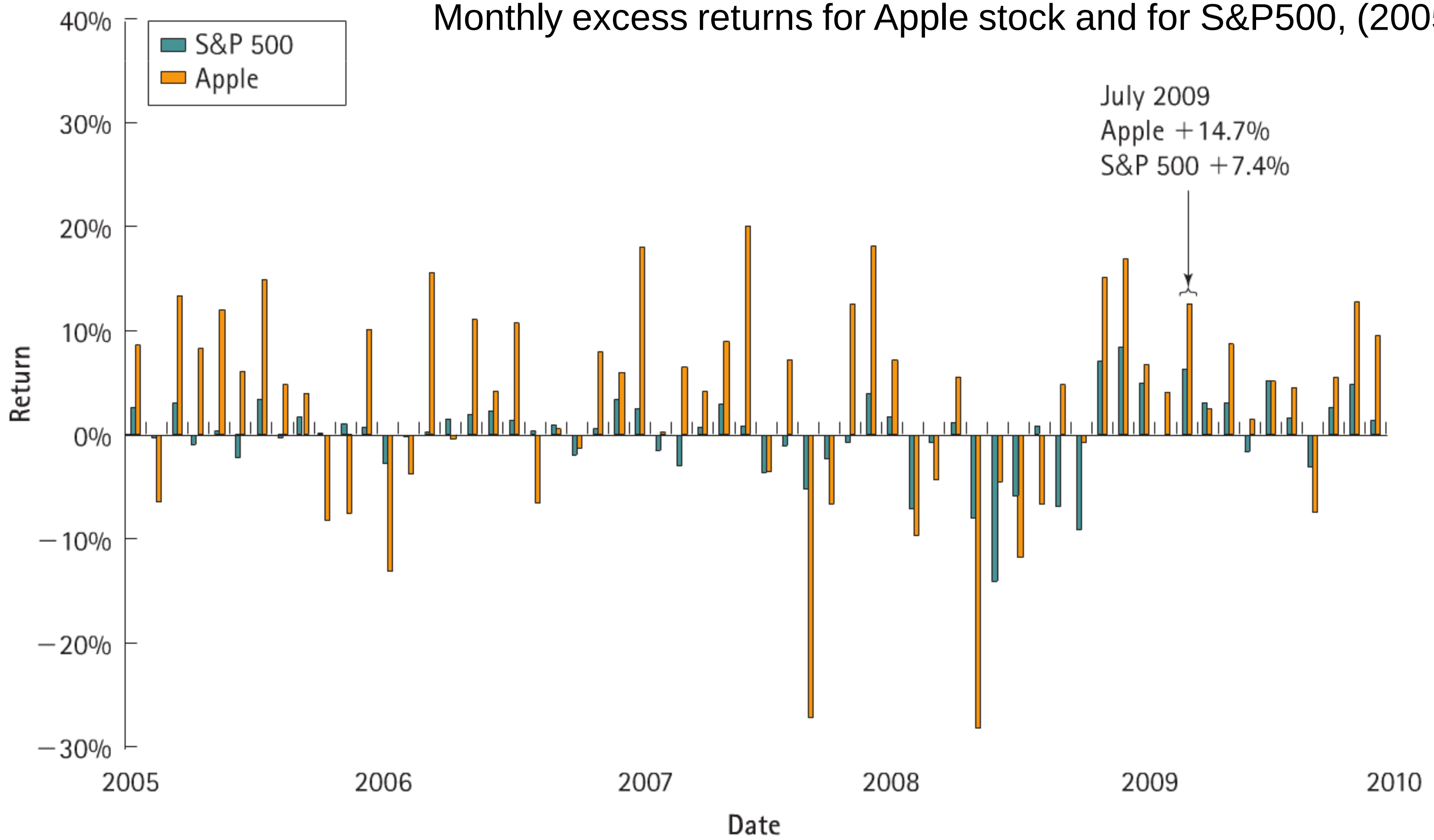
If $\beta_i < 0$ ➡ When the market as a whole stumbles, the return of i tends to rise

Interpreting Beta

Quick Check Question

Is Apple's Beta > or < to 1?

Monthly excess returns for Apple stock and for S&P500, (2005-2010)



➧ Apple's returns tend to move in the same direction than those of the market, but its movements are larger

To what extent does Beta estimations reliable?

Time Horizon: trade-off regarding which time horizon to use to measure returns

The Market Proxy: which index should we use?

Beta variation and extrapolation: Adjusted betas

$$\text{Adjusted Beta of Security } i = \frac{2}{3}\beta_i + \frac{1}{3}(1.0)$$

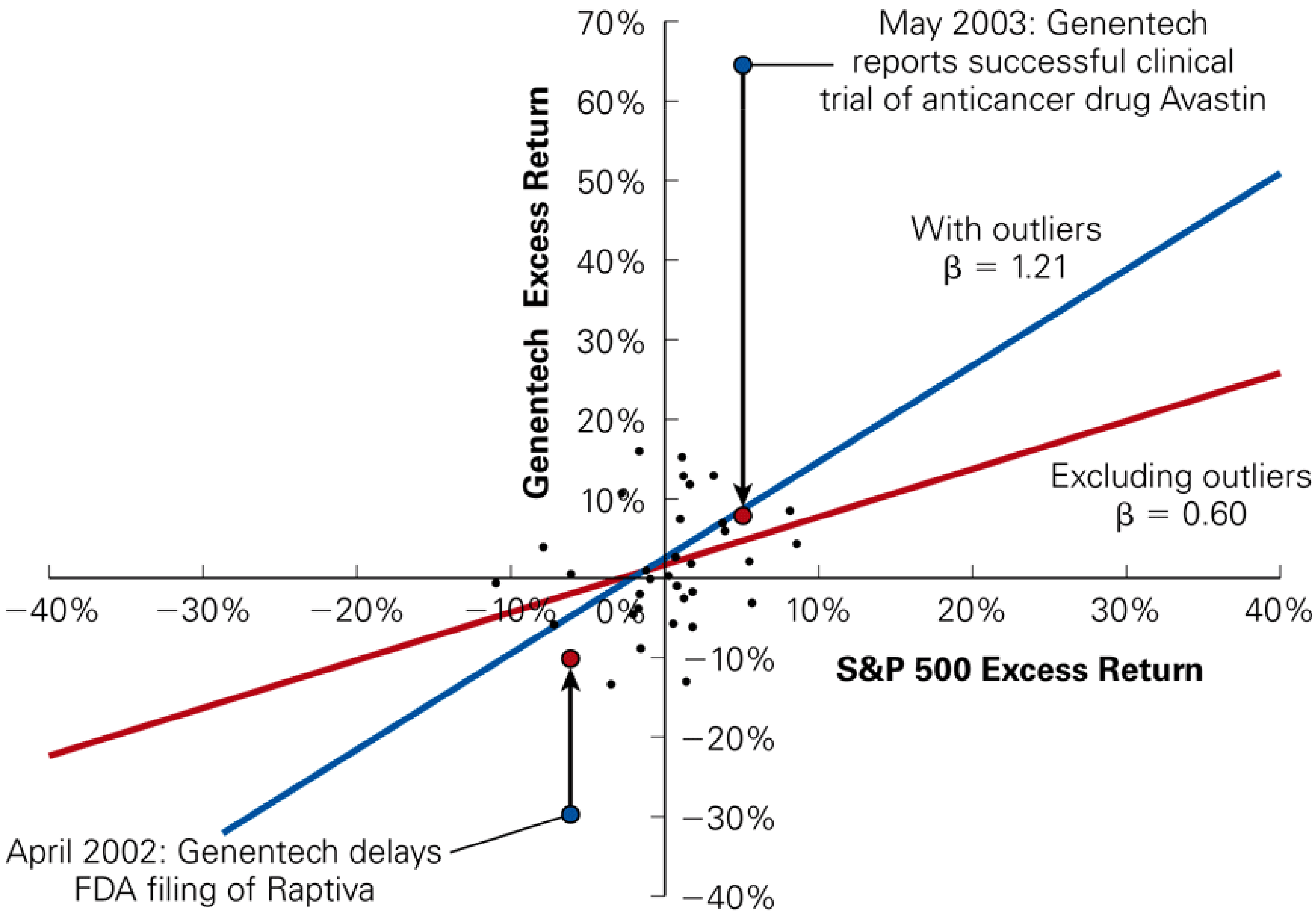
Estimation Methodologies Used by Selected Data Providers

	Value Line	Reuters	Bloomberg	Yahoo!	Capital IQ
Returns	Weekly	Monthly	Weekly	Monthly	Weekly, Monthly (5yr)
Horizon	5 years	5 years	2 years	3 years	1, 2, 5 years
Market Index	NYSE Composite	S&P 500	S&P 500	S&P 500	S&P 500 (U.S. Stocks) MSCI (International Stocks)
Adjusted	Adjusted	Unadjusted	Both	Unadjusted	Unadjusted

How to deal with outliers (returns of unusually large magnitude)?

To what extent does Beta estimations reliable?

Beta Estimation with and without Outliers for Genentech Using Monthly Returns for 2002–2004



Chapter Outline

Choosing an Efficient Portfolio	Efficient Portfolios Short Sales Efficient Frontier and diversification Identifying The Optimal Risky Portfolio
Measuring Systematic Risk: the Beta	Identifying Systematic Risk: The Market Portfolio The Beta: A measure of Systematic Risk Calculating and Interpreting Betas The Beta in practice
The Capital Asset Pricing Model (CAPM)	The CAPM Assumptions and the Market Portfolio Measuring the Cost of Capital: the CAPM Equation The Capital Market Line Versus The Security Market Line Summary of the CAPM

The CAPM : the purpose

Estimating the Equity Cost of Capital of an investment using its Beta

William F. Sharpe
Nobel prize 1990

« Capital Asset Prices. A Theory of Market Equilibrium under Conditions of Risk ».
The Journal of Finance, Vol. 19, No. 3, (1964), pp. 425-442 .

The CAPM Assumptions

Three Main Assumptions

Assumption 1

Investors can buy and sell all securities at competitive market prices (without incurring taxes or transactions costs) and can borrow and lend at the risk-free interest rate.

Assumption 2

Investors hold only efficient portfolios of traded securities—portfolios that yield the maximum expected return for a given level of volatility.

Assumption 3

Investors have **homogeneous expectations** regarding the volatilities, correlations, and expected returns of securities.

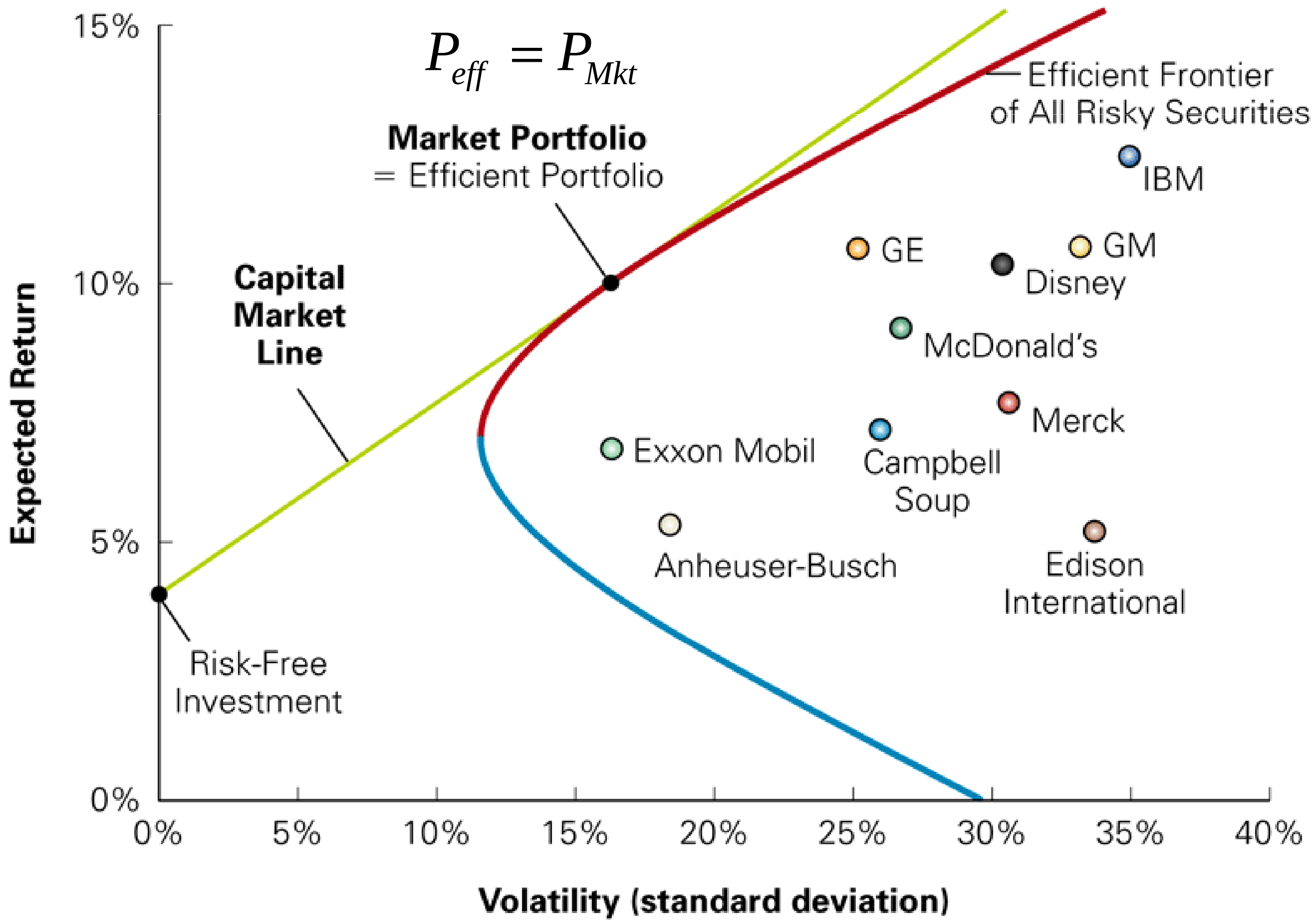
Main implication

- According to the CAPM: **The efficient portfolio is the market portfolio**

$$P_{eff} = P_{Mkt}$$

The Capital Market Line

According to the CAPM: **The efficient portfolio is the market portfolio**

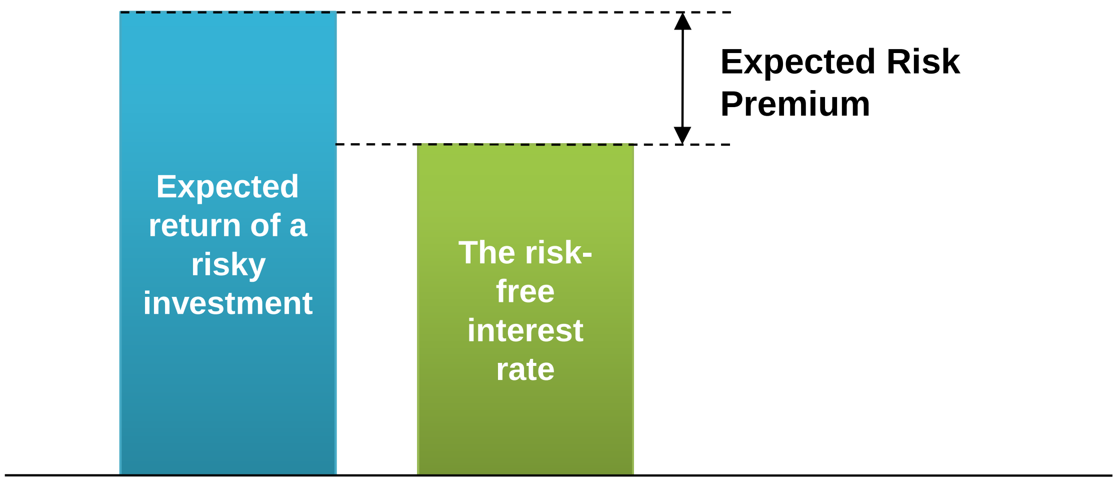


➡ the CML represents the highest expected return available for any level of volatility

Reminder

Risk Premium

Additional return (compared to risk-free interest rate) that investors expect to earn to compensate them for the security's risk



$$E[R_s] = r_f + (\text{Risk Premium of } s)$$

$E[R_s]$: Expected Return of a risky investment s

r_f : The risk-free interest rate

Market risk and Beta

$$E[R_s] = r_f + (\text{Risk Premium of } s) \quad (1)$$

When the CAPM assumptions hold:

- The risk premium of a security is determined by its Market risk (systematic risk)
- The risk premium of a security is proportional to the risk premium of the market portfolio

$$\text{Risk Premium of } s = \beta . \text{Risk Premium of } P_{\text{Mkt}} \quad (2)$$

(1) (2) $\Rightarrow$

$$E[R_s] = r_f + \beta . \text{Risk Premium of } P_{\text{Mkt}}$$

$\Rightarrow$

$$E[R_s] = r_f + \underbrace{\beta . \left(\mathrm{E}[\mathrm{R}_{P_{\text{Mkt}}}] - r_f\right)}_{\text{Risk Premium of } s}$$

The CAPM Equation

Market risk and Beta

Problem

Assume the risk-free return is 4% and market information below. **What is Renault's beta with the market? Under the CAPM assumptions, what is its expected return?**

	Expected Return	SD	Corr (RNO, Pm)
Renault (RNO)	?	35%	27%
P_{Mkt}	30%	20%	-

Market risk and Beta

Problem: A Negative Beta Stock

Assume the risk-free return is 4% and the stock of AXA has a negative beta of -0.1. **Under the CAPM assumptions, how does its expected return compare to the risk-free rate? Does this result make sense?**

	Expected Return	β
AXA	?	- 0.1
P_{Mkt}	30%	1

$$E[R_s] = r_f + \beta . (\mathrm{E}[\mathrm{R}_{P_{\mathrm{Mkt}}}] - r_f) \quad \Rightarrow \quad E[R_{AXA}] = 4\% - 0.1 * (30\% - 4\%) = 1.4\%$$

This result seems odd: why would investors be willing to accept a 1.4% expected return on this stock when they can invest in a safe investment and earn 4%?

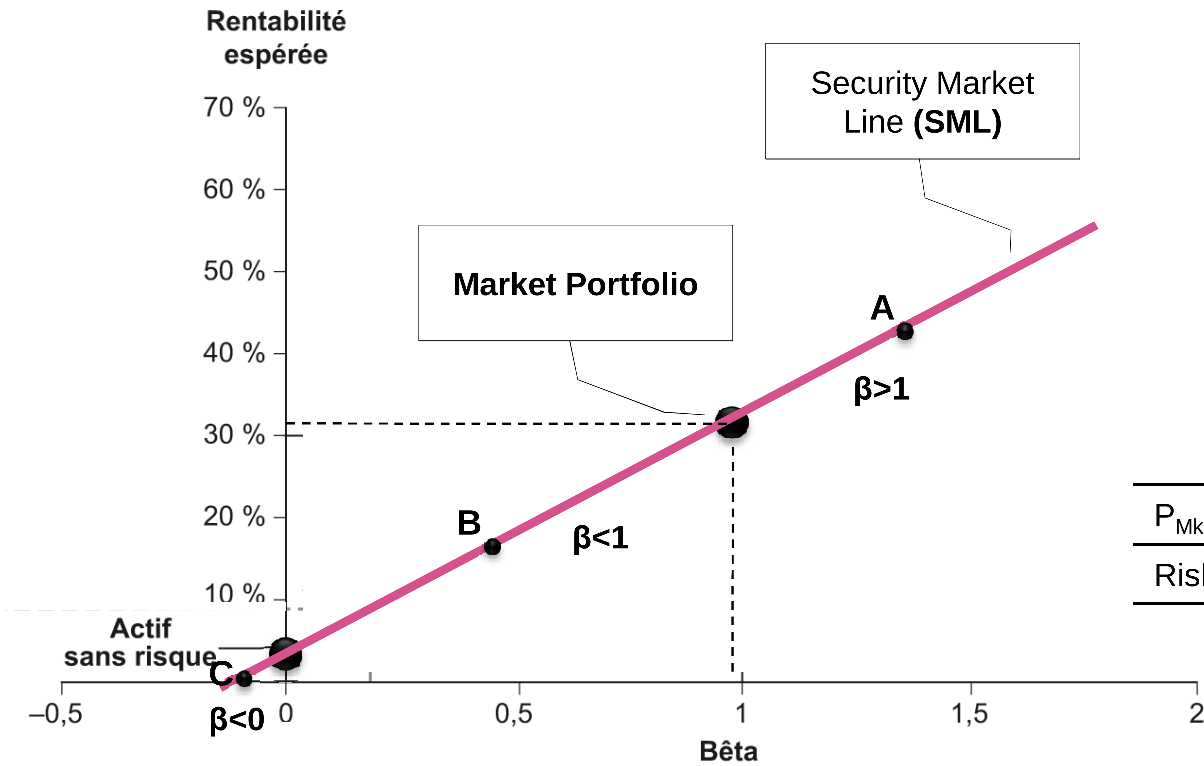
➡ A savvy investor will ………………AXA alone; instead, he will hold it in combination with other securities as part of a well-diversified portfolio

Because AXA will tend to ……….. when the market and most other securities fall, AXA provides ………………………………….for the portfolio, and investors pay for this insurance by accepting an expected return below the risk-free rate.

The Security Market Line (SML)

The SML shows the expected return for each security as a function of its beta with the market.

$$E[R_s] = r_f + \beta . (E[R_{P_{Mkt}}] - r_f)$$



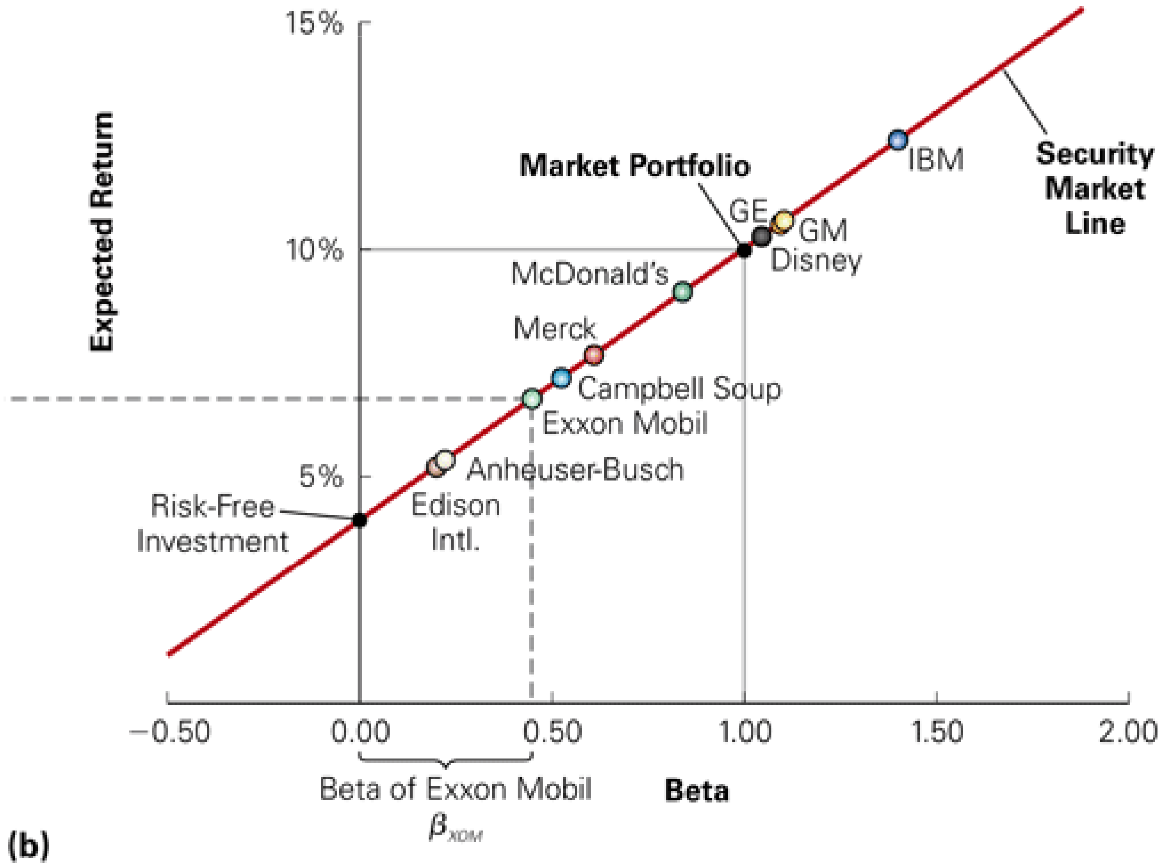
	Expected return	β
P_{Mkt}	30%	?
Risk-free Investment	4%	?

➧ According to the CAPM, the expected return of a security depends exclusively on its beta : its common risk

The Security Market Line (SML)

The SML shows the expected return for each security as a function of its beta with the market.

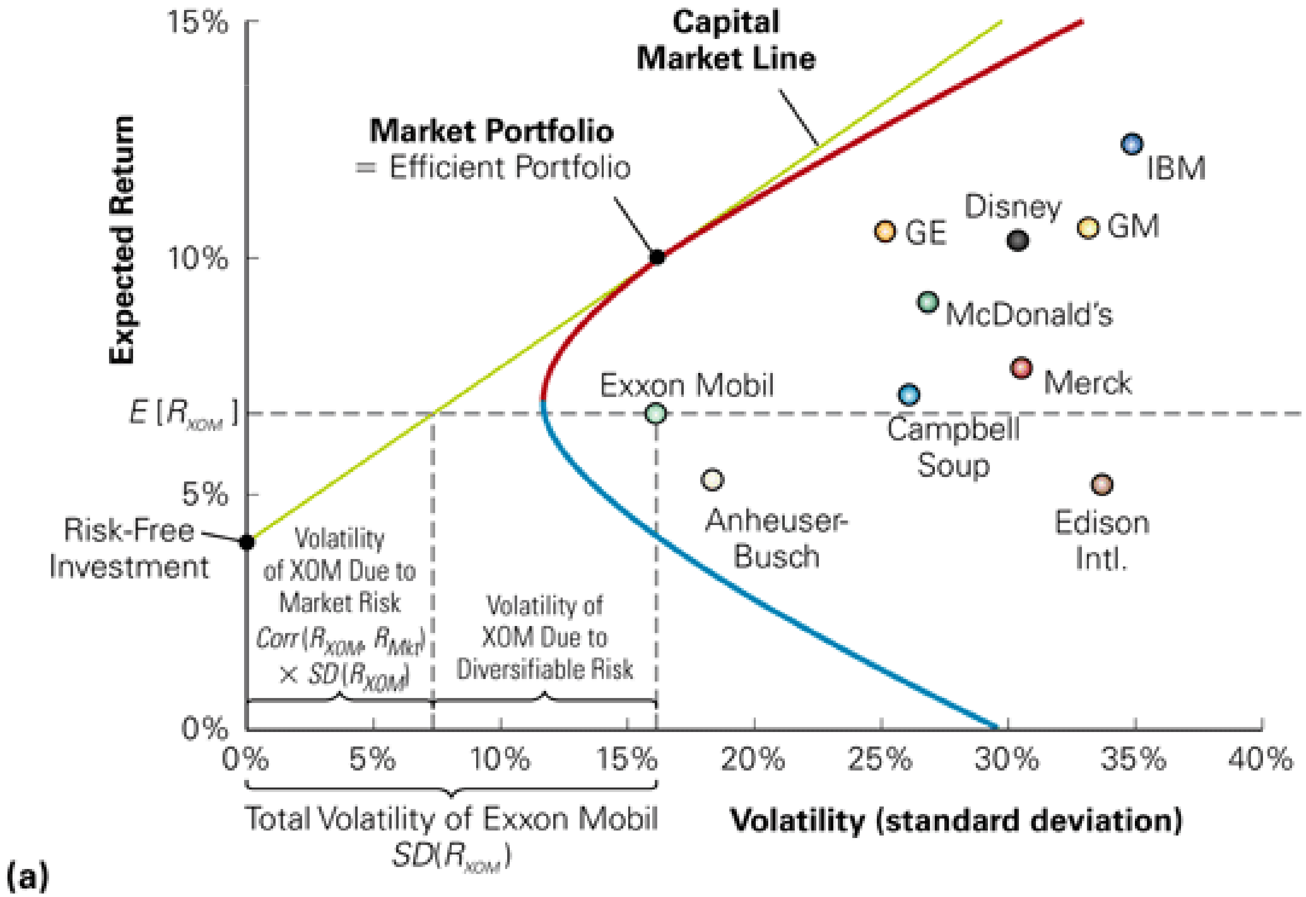
$$E[R_s] = r_f + \beta.(E[R_{P_{Mkt}}] - r_f)$$



- According to the CAPM, the market portfolio is efficient, so all stocks and portfolios should lie on the SML.

Recall : The Capital Market Line (CML)

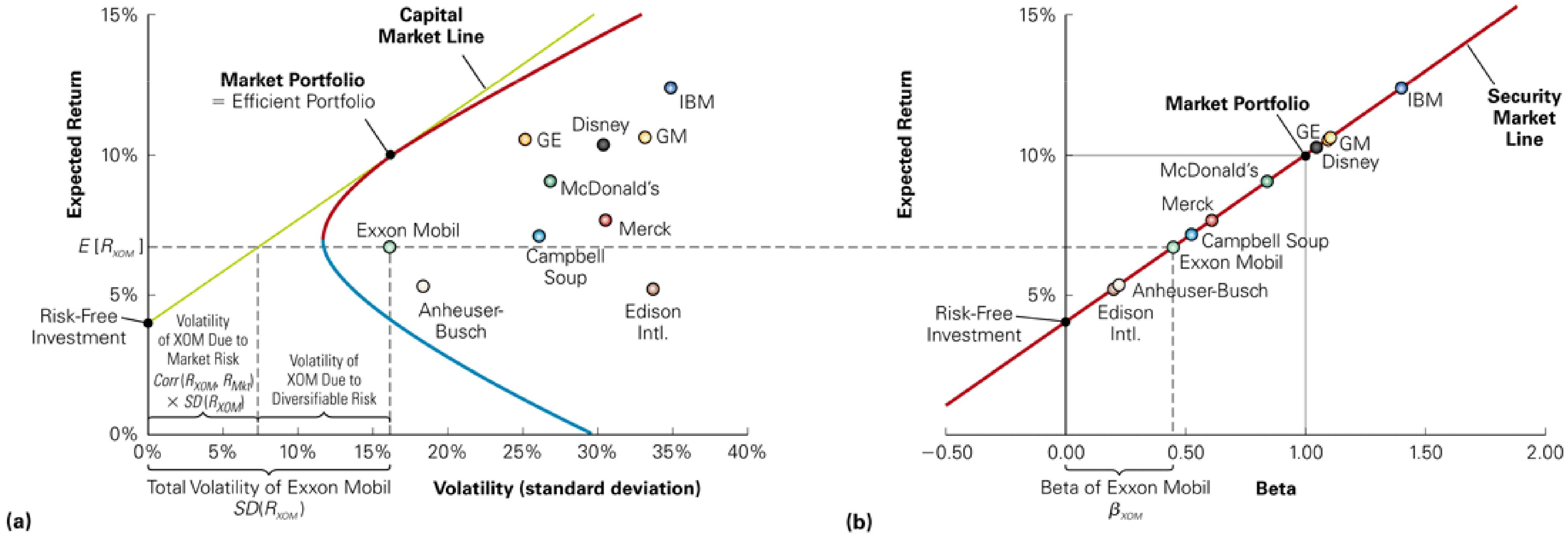
The CML depicts portfolios combining the risk-free investment and the efficient portfolio, and shows the highest expected return that we can attain for each level of volatility.



- According to the CAPM, the market portfolio is on the CML and all other stocks and portfolios contain diversifiable risk and lie to the right of the CML, as illustrated for Exxon Mobil (XOM).

The Capital Market Line Versus The Security Market Line

The Capital Market Line and the Security Market Line



$$E[R] = \text{function of its volatility } (SD)$$

$$E[R] = \text{function of its } \beta$$

- No risky security on the Capital Market Line
  - There is no relatioship between an individual stock's volatility and its expected return

The Beta of a portfolio

Beta of a Portfolio

The beta of a portfolio is the weighted average beta of the securities in the portfolio.

If $R_p = \sum_{i=1}^{N} x_i R_i$

$$\Rightarrow \quad \beta_P = \frac{Cov\left(R_P, R_{P_m}\right)}{Var\left[R_{P_m}\right]} = \frac{Cov\left(\sum_i x_i R_i, R_{P_m}\right)}{Var\left[R_{P_m}\right]} = \sum_i x_i \frac{Cov\left(R_i, R_{P_m}\right)}{Var\left[R_{P_m}\right]}$$

$$\Rightarrow \quad \boxed{\beta_P = \sum_i x_i . \beta_i}$$

The Beta of a portfolio

Problem

Assume the risk-free return is 4% and the market information below. **What is the expected return of an equally weighted portfolio of Danone and Infografe, according to the CAPM?**

	Rentabilité espérée	β
Action Danone (BN)	?	**0.5**
Action Infogrames (IFG)	?	**1.25**
P_{Mkt}	10%	

$$E[R_P] = \sum_i x_i E[R_i]$$

$$E[R_s] = r_f + \beta . \left(E[R_{P_{Mkt}}] - r_f \right)$$

$$\beta_P = \sum_i x_i . \beta_i$$

1st way

2nd way

Putting It All Together: The Capital Asset Pricing Model

According to the CAPM, the risk premium of any security is equal to the market risk premium multiplied by the beta of the security.

This relationship is called the Security Market Line (SML), and it determines the expected or required return for an investment : its equity cost of capital

The CAPM Equation

$$E[R_s] = r_f + \underbrace{\beta . \left(E[R_{P_{Mkt}}] - r_f\right)}_{\text{Risk Premium of } s}$$

With

$$\beta_i = \frac{Cov(R_i, R_{Mkt})}{[SD(R_{Mkt})]^2}$$

➡

$$\beta_i = \frac{\overbrace{SD(R_i) . Corr(R_i, R_{Mkt})}^{\text{Volatility of i that is common with the market}}}{SD(R_{Mkt})}$$

Putting It All Together: The Capital Asset Pricing Model

Recall: the discount rate to compute the NPV is the OCC

Opportunity Cost of Capital: The best available expected return offered in the market on an investment of comparable risk and term to the cash flow being discounted (Also referred to as *Cost of Capital)*

According to the CAPM, the expected return on a risky investment **only depends on its beta**

- ➡ The OCC used to compute the NPV should be the expected return on an asset that has the **same beta** of the project

Putting It All Together: The Capital Asset Pricing Model

Example : The Equity Cost of Capital

Stocks	Volatility	Beta
Arcelor Mittal	25%	1.9
Vallourec	35%	1.15

Risk-free investment	Expected Return of P_{Mkt}
4%	10%

Which stock carries more total risk?

Which has more market risk?

What is the cost of capital of an investment that has equivalent Market risk than Vallourec ?

What is the cost of capital of an investment that has equivalent Market risk than Arcelor-Mittal ?

Which investment has a higher cost of capital ?

Mittal has ……………………….cost of capital than Vallourec, even though it is less ………………………

- **Because market risk cannot be ………………………., it is ………………….. risk that determines the cost of capital**

Putting It All Together: The Capital Asset Pricing Model

The Bottom line

Under the CAPM assumptions the tangency portfolio is equal to the market portfolio

An efficient portfolio is a combination of the risk free asset and the market portfolio.

The Beta of an asset represents the risk contribution of this asset to the risk of the market.

The risk premium required by investors to hold a risky asset in their portfolio is proportional to the asset's Beta.